

Partnerships in Urban Mobility: Incentive Mechanisms for Improving Public Transit Adoption

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Abstract. *Problem definition:* Because of a prolonged decline in public transit ridership over the last decade, transit agencies across the United States are in financial crisis. To entice commuters to travel by public transit instead of driving personal vehicles, municipal governments must address the “last-mile” problem by providing convenient and affordable transportation between a commuter’s home and a transit station. This challenge raises an important question: Is there a cost-effective mechanism that can improve public transit adoption by solving the last-mile problem? *Academic/practical relevance:* In this paper, we present and analyze two incentive mechanisms for increasing commuter adoption of public transit. In a *direct* mechanism, the government provides a subsidy to commuters who adopt a “mixed mode,” which involves combining public transit with hailing rides to/from a transit station. The government funds the subsidy by imposing congestion fees on personal vehicles entering the city center. In an *indirect* mechanism, instead of levying congestion fees, the government secures funding for the subsidy from the private sector. We examine the implications of both mechanisms on relevant stakeholders. These two mechanisms are especially relevant because several jurisdictions in the United States have begun piloting incentive programs, in which commuters receive subsidies for ride-hailing trips that begin or end at a transit station. *Methodology:* We present a game-theoretic model to capture the strategic interactions among five self-interested stakeholders (commuters, public transit agency, ride-hailing platform, municipal government, and local private enterprises). *Results:* By examining equilibrium outcomes, we obtain three key findings. First, we characterize how the optimal interventions associated with the direct or the indirect mechanism depend on: (a) the coverage level of the public transit network; (b) the public transit adoption target; and (c) the relative strength of commuter preferences between driving and taking public transit. Second, we show that the direct mechanism cannot be budget-neutral without undermining commuter welfare. However, when the public transit adoption target is not too aggressive, we find that the indirect mechanism can increase both commuter welfare and sales to the private-sector partner while remaining budget-neutral. Finally, we show that, although the indirect mechanism restricts the scope of government intervention (by eliminating the congestion fee), it can dominate the direct mechanism by leaving all stakeholders better off, especially when the adoption target is modest. *Managerial implications:* Our findings offer cost-effective prescriptions for improving urban mobility and public transit ridership.

Supplemental Material: The online appendix is available at <https://doi.org/10.1287/msom.2021.0987>.

Keywords: public transit • public-private partnerships • subsidies • incentives • Mobility as a Service (MaaS)

1. Introduction

The mission of the U.S. Federal Transit Administration (FTA) is to “enhance citizens’ mobility, accessibility, and economic well-being through the development and management of public transport services.” However, over the last decade, the FTA’s mission has become increasingly threatened by a prolonged decline in public transit ridership across the United States. From 2008 to 2017, per capita ridership on buses, subways, and commuter trains saw a 4% drop in San Francisco, a 5% drop in New York City, and more than a 25% drop in both

the Los Angeles and Washington, DC, areas (The Economist 2018). Moreover, this decline is nationwide: Bus ridership has decreased from 5.6 billion trips in 2008 to 4.7 billion trips in 2018 (Kamp 2020). The widespread decline in public transit ridership is believed to be driven by multiple factors, including lower gasoline prices, changing demographic patterns within cities, and the rise of alternative modes of transportation, such as ride-hailing (Mallett 2018). As a result of declining fare revenue, more than 98% of the 2,200 public transit agencies in the United States are in financial crisis (Federal

Transit Administration 2017). Covering these losses is challenging for many municipalities due to limited funding, as well as other competing priorities, such as public safety, education, and affordable housing.

To combat declining ridership and revenue, cities can certainly scale back public transit services. However, doing so would undermine the mission of the FTA for the following reasons. Reducing access to public transit can hinder mobility, especially for those who cannot afford alternative modes of transportation (e.g., private vehicles, ride-hailing services, and taxis). Moreover, the American Public Transit Association (APTA) has found evidence suggesting that investments in public transit can generate economic returns by lifting business sales and promoting job growth (APTA 2019c). In addition to the economic benefits, public transit plays a vital role in reducing carbon emissions and alleviating traffic congestion (APTA 2019c).¹ Some cities have offered free bus rides to increase ridership (e.g., in 2019, Lawrence, Massachusetts, and Olympia, Washington, began offering free bus rides to improve mobility for the poor and the elderly (Kamp 2020)); however, the long-term financial viability of fully subsidizing bus fares is questionable. Therefore, there is an urgent need for municipalities to develop cost-effective—ideally, budget-neutral²—solutions for increasing transit ridership.

A major barrier to increasing public transit utilization is the “last-mile” problem—a challenge caused by the lack of convenient and affordable transit services between an individual’s home and a transit station (LA Metro 2016, APTA 2019b). Despite increased investments in public transit—total inflation-adjusted funding in the United States increased from 72 billion between 2007 and 2017 (APTA 2019a)—commuters continue to eschew public transit in favor of personal vehicles, in part due to the last-mile problem. Indeed, in 2017, more than 85% of U.S. workers used personal vehicles to commute, with less than 6% relying on public transit (Bureau of Transportation Statistics 2018). Further, while offering subsidized parking near transit stations may appear to be an attractive solution to address the last-mile problem, such programs are unlikely to be feasible on a large scale due to limited parking near transit stations, as well as the high cost of building, maintaining, and subsidizing parking spaces. For example, in Los Angeles County, only 24,000 subsidized parking spaces are available, to support approximately 3.6 million daily commuters (Linton 2016, U.S. Census Bureau 2018).

To address the last-mile problem, various municipal governments are forming partnerships with private transportation companies. For example, in 2019, Los Angeles partnered with the ride-hailing platform Via to launch a pilot project that offers commuters subsidized rides to and from public transit stations. The

subsidy is supported by a grant from the Federal Transit Administration (Los Angeles County Metropolitan Transportation Authority 2019). Several other major cities (e.g., Atlanta; Austin, Texas; Detroit; Philadelphia; and Tampa, Florida) have also begun testing the use of ride-hailing subsidies as a potential solution to closing the last-mile gap (Schwieterman and Livingston 2018, APTA 2019e). These subsidy programs have only recently become possible due to advances in information technology, which now enables on-demand ride-hailing, real-time tracking of passenger locations, and online/mobile fare payments.³ More generally, the integration between public transit systems and ride-hailing is accelerating—for example, in early 2020, Uber announced a new in-app feature that coordinates trip drop-off times with train schedules (Uber 2020). This novel integration between private transportation services and public transit opens the door to new forms of mutually beneficial partnerships.⁴

In this paper, we investigate two incentive mechanisms that aim to increase public transit ridership by addressing the last-mile problem. Both mechanisms exhibit the following characteristics:

1. **Both mechanisms have a common feature.** Both mechanisms rely on a strategic partnership between a public transit agency and a private ride-hailing platform. Specifically, in both mechanisms, commuters receive a subsidy for adopting a *mixed mode* of transportation, in which commuters use the ride-hailing service to travel the last-mile distance (“ x ”) between their homes and a transit station and use public transit to travel between the transit station and a final destination (e.g., city center). In this setting, x can be interpreted as a measure of the *coverage*⁵ of the public transit system: The last mile x is small (large) if commuters have convenient (inconvenient) access to transit stations from their homes.
2. **Both mechanisms share a common goal.** Both mechanisms are intended to increase the adoption of the mixed mode by a constant factor (“ β ”).

The key differences between the mechanisms are: (a) the role of the local government; and (b) the source of funding for the ride-hailing subsidy. To elaborate, the first incentive program we consider is *direct* in the sense that the government offers a subsidy (“ s ”) directly to each commuter as an incentive for adopting the mixed mode. To defray the cost associated with the subsidy, the government charges a congestion fee (“ e ”) to commuters who instead choose to drive a personal vehicle into the city.⁶ Therefore, under the “direct mechanism” (denoted by [D]), the government uses two levers to entice commuters to adopt the mixed mode: the ride-hailing subsidy s and the congestion fee e .

Unlike the direct mechanism [D] that relies on congestion fees collected from commuters who drive, the

local government secures funding for the subsidy (“ z ”) from the private sector in the second mechanism. This mechanism is *indirect* because the government’s role is restricted to facilitating the transfer of the subsidy from a private enterprise to commuters. This scheme is motivated by the fact that many municipalities may not have sufficient funds to further subsidize public transit. To relieve this financial burden, Cole (2017) proposes the following partnership between a municipal government and a private enterprise. The government and public transit agency develop a mobile app to process different transactions, including fares collected from the commuters and subsidies provided by the private enterprise. The private enterprise benefits from higher transit ridership through an expanded customer base. Here, the customer base may represent foot traffic at stores near the transit station (e.g., coffee shops, bakeries, health clinics, spas, or hair salons) or number of views on an in-app advertisement. Under this arrangement, the private enterprise can recover the cost associated with the subsidy from the extra revenue derived from the increased demand. In contrast to the direct mechanism, this “indirect mechanism” (denoted by [I]) involves just a single lever: the subsidy z , provided by the private enterprise.

Mechanism [I] is motivated by a growing interest among transit agencies in leveraging private-sector partnerships to subsidize public transit trips. For example, since early 2020, commuters in Miami-Dade County, Florida, can earn loyalty points (i.e., subsidies) by taking public transit, which can be redeemed for discounts on future trips through the transit agency’s mobile app (known as “Go Miami-Dade Transit”) (Corselli 2020). These loyalty points are funded by local businesses, who post in-app video ads within the mobile app. The National Hockey League (NHL) and the Seattle Monorail formed a partnership in which the NHL will subsidize rides on the Monorail for fans attending hockey games (Condor 2020). Both partnerships can be viewed as a private enterprise (i.e., a single aggregate entity) subsidizing public transit trips in exchange for an expanded audience, which captures the main characteristics of mechanism [I].

1.1. Outline and Contributions

Our intent is to investigate the impact of both subsidy mechanisms on multiple stakeholders: a municipal government, suburban commuters, city dwellers, a public transit agency, a ride-hailing platform, and a private enterprise. To do so, we examine the following questions:

1. How does each mechanism perform in terms of: (a) operating cost (borne by the government); (b) commuter welfare; (c) the public transit agency’s revenue; (d) the ride-hailing platform’s revenue; and (e) the private enterprise’s profit?

2. How does the last-mile distance (or coverage) x and the mixed mode adoption target β affect: (a) the optimal direct subsidy s^* and congestion fee e^* in mechanism [D]; and (b) the optimal indirect subsidy z^* in mechanism [I]?

3. Under what conditions, if any, should the municipal government adopt mechanism [I] over mechanism [D]?

We examine the above questions by developing a game-theoretic model that captures the interactions among all stakeholders. Specifically, in mechanism [D], we identify the optimal subsidy level s^* and congestion fee e^* that minimizes the government’s net spending (subsidy cost less the revenue generated by the congestion fee), subject to improving adoption of the mixed mode by a factor of at least β . In addition, we restrict the set of feasible subsidies and congestion fees to those that satisfy participation constraints associated with commuters, the ride-hailing platform, the public transit agency, and the private enterprise. We also determine the optimal subsidy z^* under mechanism [I], subject to the same set of stakeholder participation constraints.

Our key findings are summarized as follows:

1. **The optimal incentive under mechanism [D] depends on the last-mile distance x .** Specifically, the optimal incentive entails a large congestion fee and a small ride-hailing subsidy when the transit coverage is high (i.e., when the last-mile distance x is small). Conversely, when transit coverage is low (i.e., x is large), this prescription is reversed: The optimal incentive relies on aggressive ride-hailing subsidies and relatively smaller congestion fees. Moreover, we find that whether the optimal subsidy s^* increases or decreases in the last-mile distance x depends on the commuter’s relative preference between driving and taking public transit.

2. **Budget-neutrality is not attainable under mechanism [D]:** It is not possible for the government to fully recoup the cost of the subsidy through congestion fees without decreasing total commuter welfare.

3. **Mechanism [I] can dominate mechanism [D] under certain conditions.** Despite the government having access to two levers in mechanism [D] (the subsidy s and the congestion fee e) and only a single lever in mechanism [I] (the subsidy z), mechanism [I] can *dominate* mechanism [D], but only when the adoption target β is modest. Specifically, when β is modest, the commuters, city dwellers, public transit agency, ride-hailing platform, and private enterprise are all better off under mechanism [I].

The remainder of the paper is organized as follows. In Section 1.2, we discuss related literature. In Section 2, we present a general model for analyzing both mechanisms. In Section 3, we present mechanism [D] and analyze the optimal incentive scheme (e^*, s^*). In Section 4, we present mechanism [I], analyze the optimal subsidy z^* , and compare the performance of both mechanisms. In Section 5, we consider variants of mechanisms [D] and [I] that address the trade-off between operating cost and commuter welfare. In Section 6, we conclude

and discuss potential future research directions. All proofs are contained in the online appendix.⁷

1.2. Related Literature

Our paper is related to three streams of literature: government subsidy programs, public-private partnerships, and budget-neutral mechanisms.

1.2.1. Government Subsidy Programs. Within the operations literature, there is a growing focus on subsidy programs that promote the production or adoption of socially beneficial goods or services. This work can be categorized into subsidies for producers (e.g., farmers, manufacturers, or healthcare providers) and consumers. Subsidies for producers have been studied in the context of healthcare (Taylor and Xiao 2014, Levi et al. 2017, Aswani et al. 2019), green technology (Bansal and Gangopadhyay 2003, Alizamir et al. 2016, Ma et al. 2019), and agriculture (Alizamir et al. 2019, Akkaya et al. 2021). Our paper is closer to the literature on consumer-facing subsidies. In the home appliances industry, Yu et al. (2018) determine whether the government should subsidize consumers only, manufacturers only, or both, where the goals of the government are to improve manufacturer profit and consumer welfare in rural areas. Xiao et al. (2020) examine the impact of the same home-appliance subsidy program, which they conclude improves both affordability and accessibility for rural customers. In the context of solar energy technology, Chemama et al. (2019) compare the effectiveness of static and dynamic consumer subsidies on supplier behavior. Our paper differs from this existing literature in that we investigate whether the subsidy program can be budget-neutral from the perspective of the subsidy provider.

Previous work in the operations literature on public transit subsidies is sparse. Lodi et al. (2016) consider a setting where operation of the public transit system is outsourced to the private sector, and the government provides a subsidy to offset the operating cost. Yang and Lim (2018) use a field experiment to show that temporarily subsidizing public transit can lead to long-term changes in commuter behavior. The paper that is most similar to ours in this line of research is by Xiao and Zhang (2014), who also consider congestion fees and transit subsidies simultaneously. They focus on the impact that commuter heterogeneity in value-of-time has on the optimal design of congestion fees and show that transit subsidies can offset the loss in commuter welfare due to the congestion fee. Our work is different in that we focus on the role that congestion fees and transit subsidies can play in improving public transit ridership.

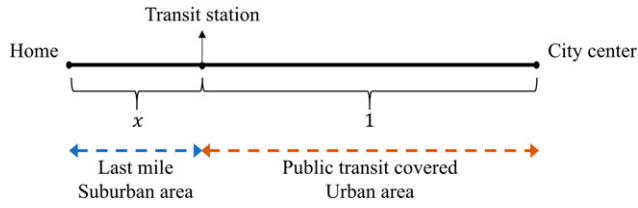
1.2.2. Public-Private Partnerships. Our work contributes to the modeling of public-private partnerships (PPPs), which refer to a private-sector partner “financing, constructing, and managing a project in

return for a promised stream of payments directly from government or indirectly from users over the projected life of the project or some other specified period” (Weimer and Vining 2017, p. 309). Our paper belongs to the stream of PPP literature where the government has all of the bargaining power. In the existing literature, this is often modeled as either a principal-agent problem or a Stackelberg game, where the government is the principal/leader and the firm is the agent/follower; applications include healthcare (So and Tang 2000, Lee and Zenios 2012, Gupta and Mehrotra 2015, Aswani et al. 2019, Guo et al. 2019), disaster management (Guan and Zhuang 2015, Guan et al. 2018), and risk management (Bakshi and Gans 2010). With respect to transportation, existing work on public-private partnerships has primarily focused on infrastructure. Lodi et al. (2016) address the government’s incentive-design problem when management of the public transit service is outsourced to private operators. Gagnepain and Ivaldi (2002) examine the effect on social welfare under different types of contracts between the regulator and the public transit operator. Hansson (2010) considers a multiprincipal setting, where the local, regional, and county governments interact to regulate public transit procurement. In contrast to these papers, the private partners play a different role in our work—namely, they provide a complementary transportation service (in the case of the ride-hailing platform) and funding for subsidies (in the case of the partner enterprise).

Previous work has also considered settings where the public-private partnership is formed through negotiation. In the transportation context, Kang et al. (2013) investigate royalty bargaining associated with underground railway station construction, and Wang and Zhang (2016) examine road pricing of transportation networks with both public and private roads. Other application areas include natural resource development (Anandalingam 1987), public procurement (Gur et al. 2017, Saban and Weintraub 2021), and global supply chain management (Cohen et al. 2018, Cho et al. 2019, Cohen and Lee 2020).

1.2.3. Budget-Neutral Mechanisms. This paper also contributes to the literature on budget-neutral policies. This work has primarily appeared in the public policy literature and has addressed issues such as Social Security reform (Burkhauser and Smeeding 2011), environmental taxation (Goulder 1995, Murray and Rivers 2015), and fiscal policy (Correia et al. 2013, D’Acunto et al. 2016). Within the operations management literature, previous work on budget-neutral policies is scarce. Guo et al. (2014) optimize a two-tier queuing system with both a free server and fee-based server, in the setting where the system is self-financed by the costly server. Arifoglu and Tang (2021) develop

Figure 1. (Color online) Travel Distance of a Commuter



a budget-neutral incentive mechanism for coordinating a decentralized influenza vaccine supply chain.

With respect to transportation settings, most existing papers that focus on budget-neutrality are based on schemes under which subsidies/rebates are funded by congestion fees, similar to mechanism [D] in our work (see, e.g., Guo and Yang 2010, Nie and Liu 2010, Chen and Yang 2012, and Xiao and Zhang 2014). Our paper differs in that we also consider an indirect mechanism [I], where the government attains budget-neutrality by obtaining funding from a private enterprise, instead of imposing congestion fees. Further, we compare mechanisms [D] and [I] in terms of their impact on the relevant stakeholders.

2. Model Preliminaries

A unit mass of commuters are located a (last mile) distance $x > 0$ from a transit station. All commuters must travel to a city center that is located an additional distance of one beyond the transit station. For tractability, we consider a parsimonious model where commuters choose between two modes of travel:⁸ *driving* or a *mixed mode*. As depicted in Figure 1, x represents the length of the last mile that is not covered by public transit. Hence, commuters who choose the driving mode will drive a personal vehicle for a distance of $1 + x$ to the city center. However, commuters who choose the mixed mode will first travel a distance of x to the transit station via a ride-hailing service and then travel the remaining distance of one by public transit.

2.1. Base Model

We first describe our base model in the absence of any subsidies or congestion fees. In later sections, we extend our base model to incorporate the two incentive schemes. We note here that our focus throughout the paper is on daily commuters who live in suburban areas outside the coverage area of the public transit system; therefore, all aspects of the model (e.g., transit ridership, operating cost, and ride-hailing platform revenue) are defined with respect to this group of commuters only.

2.1.1. Commuter Utility. Each commuter obtains a “base” utility M for commuting to the city center. A commuter has valuation (or willingness-to-pay) V per

unit distance traveled via ride-hailing or public transit, or δV if she drives.⁹ We assume $\delta > 1$ to reflect a higher intrinsic utility for driving.¹⁰ To capture heterogeneity among commuters, we assume $V \sim U[0, 1]$.¹¹

A commuter will travel a distance of $1 + x$ if she chooses the driving mode of transportation. Similarly, under the mixed mode, the commuter will first travel a distance of x via ride-hailing and the remaining distance of one via public transit. In our model, we assume a commuter’s cost (or fare) for driving, ride-hailing, and taking public transit are denoted by d , r , and p , respectively. We assume throughout that $r > d > p > 0$, which is supported empirically.¹² Therefore, the utilities associated with driving and the mixed mode in the absence of any incentives, denoted by U_d^0 and U_m^0 , respectively, are given by

$$U_d^0 = M + (x + 1)\delta V - (x + 1)d, \quad (1a)$$

$$U_m^0 = M + (x + 1)V - (rx + p). \quad (1b)$$

2.1.2. Travel-Mode Demand. It follows from (1) that a commuter will adopt the mixed mode if and only if $U_m^0 \geq U_d^0$. We assume the base value M is large enough such that $U_d^0 \geq 0$ and $U_m^0 \geq 0$. Therefore, $U_m^0 \geq U_d^0$ if and only if:

$$V \leq v^0 \equiv \frac{(d - p) - (r - d)x}{(\delta - 1)(x + 1)}. \quad (2)$$

Let D_d^0 and D_m^0 denote the demand for driving and the mixed mode, respectively. Because $V \sim U[0, 1]$, we can apply (2) to show that:

$$D_d^0 = \int_{v^0}^1 1 \cdot dV = 1 - \frac{(d - p) - (r - d)x}{(\delta - 1)(x + 1)}, \quad (3a)$$

$$D_m^0 = \int_0^{v^0} 1 \cdot dV = \frac{(d - p) - (r - d)x}{(\delta - 1)(x + 1)}. \quad (3b)$$

Note that D_m^0 represents the additional public transit ridership generated by the mixed-mode commuters. Because $r > d > p > 0$, it is straightforward to verify that D_m^0 is decreasing in x , which implies that commuters that live far from the transit station are more likely to drive and less likely to adopt the mixed mode. Further, D_m^0 also measures the “reduction” in traffic congestion, because $D_m^0 = 1 - D_d^0$.

Let $[a]^+ = \max\{a, 0\}$. To exclude the degenerate cases where *all* commuters adopt the same travel mode, we assume that $x \in (\underline{x}, \bar{x})$, so that $D_m^0 > 0$ and $D_d^0 > 0$, where $\underline{x} = \left[\frac{(d-p) - (\delta-1)}{(\delta-1) + (r-d)} \right]^+$ and $\bar{x} = \frac{d-p}{r-d}$. We also assume that the public transit and the ride-hailing platform have sufficient capacity to accommodate demand for the mixed mode.¹³

2.1.3. Commuter Welfare. Because $V \sim U[0, 1]$, the commuter welfare in the base model is given by

$$W^0 = \int_0^{v^0} U_m^0 dV + \int_{v^0}^1 U_d^0 dV. \quad (4)$$

2.1.4. Transit-Agency Revenue. By noting that the unit fare of public transit is p , the revenue generated by the public transit agency from these commuters under the mixed-mode demand D_m^0 is:

$$\Pi_p^0 = p \cdot D_m^0. \quad (5)$$

2.1.5. Ride-Hailing Platform Revenue. Recall that each mixed-mode commuter will travel a distance of x via ride-hailing to the transit station, at a unit fare of r . The revenue generated by the ride-hailing platform from these commuters under the mixed-mode demand D_m^0 is:

$$\Pi_r^0 = r \cdot x \cdot D_m^0. \quad (6)$$

2.1.6. Private Enterprise Profit. Consider a private enterprise who sells to public transit commuters. This private enterprise may be a business that sells goods or services to passengers that pass through a transit station, or one that places advertisements within a transit system's mobile app or physical walkways. We model the enterprise profit as $(k - c)[1 - k\alpha]^+ D_m^0$, where k and c represent the unit price and cost, respectively, and $[1 - k\alpha]^+$ represents the proportion of mixed-mode commuters that purchase the product. Note that $\alpha > 0$ is the commuters' price sensitivity. To avoid the trivial case where the enterprise's optimal profit is nonpositive, we assume $\alpha c < 1$.¹⁴ Then, for any given mixed-mode demand D_m^0 , the optimal retail price in the base model is $k_b = \frac{\alpha c + 1}{2\alpha}$, so that the corresponding optimal profit is equal to $(k_b - c)[1 - k_b\alpha]^+ D_m^0 = \frac{(1 - \alpha c)^2}{4\alpha} D_m^0$. For ease of exposition, we let $K \equiv \frac{(1 - \alpha c)^2}{4\alpha}$ denote the profit margin, so that the corresponding total profit can be expressed as:

$$\Pi_s^0 = K \cdot D_m^0. \quad (7)$$

2.2. A Generalized Mechanism: Government Subsidy, Congestion Fees, and Private Subsidy

By using the quantities as defined above, we now present our model that incorporates the aforementioned incentives. To avoid repetition, we shall present a generalized mechanism that combines three interventions: (1) a congestion fee e charged to each commuter who travels by a personal vehicle; (2) a subsidy s for each commuter who takes the mixed mode, paid for by the government; and (3) a subsidy z for each commuter who takes the mixed mode, paid for by the private

enterprise. We denote this generalized mechanism by (e, s, z) . When $z = 0$, this mechanism reduces to mechanism [D]. Also, when $e = s = 0$, the mechanism simplifies to mechanism [I]. To isolate the effect of (e, s) under mechanisms [D] and the effect of s under mechanism [I], we shall analyze mechanism [D] in Section 3 and mechanism [I] in Section 4 as two separate mechanisms.

The sequence of events is as follows. First, the government chooses whether to adopt mechanism [D] or mechanism [I]. Second, the government sets the incentives that correspond to the chosen mechanism. Specifically, the government sets the congestion fee e and government subsidy s if mechanism [D] is chosen, or the private subsidy z if mechanism [I] is chosen. Next, once (e, s) or z is set, the private enterprise selects the unit price k to maximize its profit. Finally, each commuter chooses to commute by either driving or adopting the mixed mode.

By considering (1) along with the incentive (e, s, z) , commuter utilities associated with different modes of transportation can be written as:

$$U_d(e, s, z) = U_d^0 - e, \quad (8a)$$

$$U_m(e, s, z) = U_m^0 + s + z, \quad (8b)$$

where U_d^0 and U_m^0 are given in (1).¹⁵

Hence, a commuter will adopt the mixed mode under incentive (e, s, z) if and only if $U_m(e, s, z) \geq U_d(e, s, z)$, or, equivalently, if her valuation V satisfies:

$$V \leq v(e, s, z) \equiv \min\left\{1, \frac{(d - p) - (r - d)x + e + s + z}{(\delta - 1)(x + 1)}\right\}. \quad (9)$$

Akin to the base case without incentives, the demand for each transportation mode under the incentive (e, s, z) is given by

$$D_d(e, s, z) = \left[1 - \frac{(d - p) - (r - d)x + e + s + z}{(\delta - 1)(x + 1)}\right]^+, \quad (10a)$$

$$D_m(e, s, z) = 1 - \left[1 - \frac{(d - p) - (r - d)x + e + s + z}{(\delta - 1)(x + 1)}\right]^+. \quad (10b)$$

Note that if $(e, s, z) = (0, 0, 0)$, then $v(e, s, z)$, $D_d(e, s, z)$, and $D_m(e, s, z)$ reduce to their base values v^0 , D_d^0 , and D_m^0 , as stated in Section 2.1, respectively.

2.2.1. Performance Metrics. Next, we define the metrics by which we evaluate the performance associated with the incentive (e, s, z) . Because the government offers a subsidy s to each of the $D_m(e, s, z)$ mixed-mode commuters and collects a fee e from each of the $D_d(e, s, z)$ commuters who drive, and meanwhile the government passes on a subsidy z from the partner enterprise to each mixed-mode commuter, the total cost for the government to operationalize the incentive (e, s, z) is:

$$C(e, s, z) = (s + z)D_m(e, s, z) - eD_d(e, s, z) - zD_m(e, s, z) = sD_m(e, s, z) - eD_d(e, s, z). \quad (11)$$

Note that when $e = s = 0$, there is no operating cost $C(0, 0, z) = 0$. Similar to the base case, the total welfare accrued to all commuters is:

$$W(e, s, z) = \int_0^{v(e, s, z)} U_m(e, s, z) dV + \int_{v(e, s, z)}^1 U_d(e, s, z) dV, \quad (12)$$

the public transit agency's revenue generated from these commuters is:

$$\Pi_p(e, s, z) = p \cdot D_m(e, s, z), \quad (13)$$

and the ride-hailing platform's revenue generated from these commuters is:

$$\Pi_r(e, s, z) = r \cdot x \cdot D_m(e, s, z). \quad (14)$$

Lastly, the private enterprise's profit generated from these commuters is $((k - c)[1 - k\alpha]^+ - z)D_m(e, s, z)$. Observe that the optimal price $k_g = \frac{ac+1}{2\alpha}$, so that the corresponding optimal profit is $(K - z)D_m(e, s, z)$, where K is the enterprise's profit margin in the base model, as defined in Section 2.1. We assume the profit margin of a participating enterprise K satisfies $K > \underline{K}$ throughout the paper, where $\underline{K} = (d - p) - (r - d)x$, because enterprises with low profit margin would not opt in to the subsidy program. Hence, the enterprise's total profit under incentives (e, s, z) is

$$\Pi_s(e, s, z) = (K - z) \cdot D_m(e, s, z). \quad (15)$$

Note that when $(e, s, z) = (0, 0, 0)$, the metrics $W(e, s, z)$, $\Pi_p(e, s, z)$, $\Pi_r(e, s, z)$, and $\Pi_s(e, s, z)$ reduce to the base values W^0 , Π_p^0 , Π_r^0 , and Π_s^0 (defined in Section 2.1), respectively.

Next, we present the evaluation criteria for the incentive program (e, s, z) . In particular, we assume that (e, s, z) is set according to the following criteria:

I. **Minimum operating cost.** Because most municipalities in the United States and around the world are budget-constrained with respect to public transit,¹⁶ we assume that the government has a strong desire to minimize the total operating cost of the incentive program, but not profit from it, where the government's operating cost is given by $[C(e, s, z)]^+$.

II. **Increase transit ridership.** The intent of the incentive (e, s, z) is to increase public transit ridership among commuters by a factor of $\beta > 0$. We assume throughout that β is an exogenous adoption target and that the incentive (e, s, z) must satisfy:

$$D_m(e, s, z) - D_m^0 \geq \beta D_m^0,$$

where D_m^0 is the baseline mixed-mode adoption without any intervention, as stated in Section 2.1. Note that the increase β is equivalent to a decrease in demand for driving; therefore, β may be

equivalently interpreted as the target reduction in traffic congestion. For this reason, the constraint above can also be interpreted as improving the well-being of city dwellers. Analogous to the assumption that $x \in (\underline{x}, \bar{x})$ in the base model, we assume throughout that the target β is restricted to the nondegenerate case, where $D_d(e, s, z) > 0$ and $D_m(e, s, z) > 0$. To enforce the preceding inequalities, it suffices to assume that $\beta < \bar{\tau}(x) \equiv \frac{(\delta-1)(x+1)}{(d-p)-(r-d)x} - 1$. Note that if $\beta \geq \bar{\tau}(x)$, then upon implementation of incentive (e, s, z) , no commuters will drive ($D_d(e, s, z) = 0$), which is unlikely to occur in practice.

III. **Commuter participation.** To generate public support for the incentive program, commuters should not be worse off collectively: $W(e, s, z) \geq W^0$.

IV. **Ride-hailing platform participation.** To form a viable partnership with the ride-hailing platform, the platform's revenue generated from daily commuters should not decrease: $\Pi_r(e, s, z) \geq \Pi_r^0$.

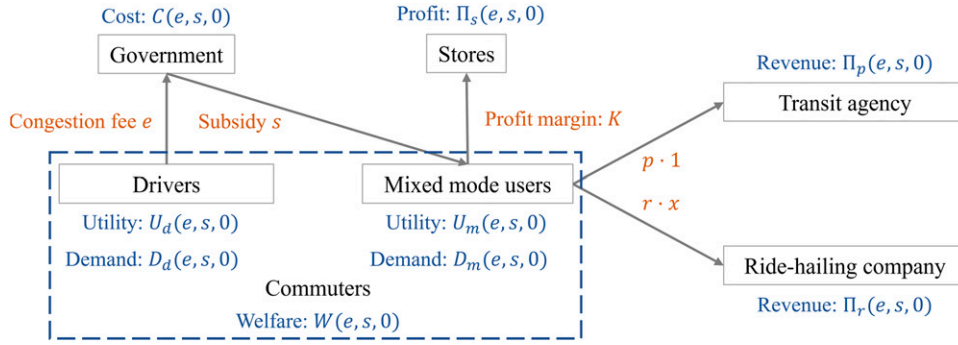
V. **Public transit participation.** To ensure that the public transit agency is not negatively impacted, its revenue generated from the mixed mode should not decrease: $\Pi_p(e, s, z) \geq \Pi_p^0$.¹⁷

VI. **Enterprise participation.** Lastly, to prevent adverse effects on the local economy, it is desirable to ensure that the incentive (e, s, z) does not reduce the private enterprise's profit: $\Pi_s(e, s, z) \geq \Pi_s^0$. Further, in mechanism [I] in particular, to entice participation of the private enterprise, the private subsidy z is set to maximize the private enterprise's profit: $z = \operatorname{argmax}_z \Pi_s(e, s, z)$.

Observe from criterion (II) that a feasible incentive program (e, s, z) must satisfy $D_m(e, s, z) \geq D_m^0$. Combine this observation with (13) and (14), and criteria (IV) and (V) are satisfied (and, hence, redundant) immediately if $D_m(e, s, z) \geq D_m^0$. This result is intuitive: When the government increases the mixed-mode adoption, both the ride-hailing company and the transit agency will be better off. Therefore, we shall omit criteria (IV) and (V) from the incentive-design problems in the remainder of the paper for conciseness.

3. Direct Mechanism: Commuter Subsidies and Congestion Fees

In the direct mechanism [D], the local government directly offers a subsidy s to each mixed-mode commuter and charges a congestion fee e to each commuter who drives. Hence, by letting $z = 0$ in the general formulation presented in Section 2.2, we can determine the commuter utility and the market demand associated with the driving mode and the mixed mode, along with all the performances metrics under mechanism [D]. Figure 2 depicts the relationship among the various stakeholders in this case.

Figure 2. (Color online) Strategic Interactions Among Stakeholders Under the Direct Mechanism [D]

Based on the evaluation criteria (I)–(VI) (with (IV) and (V) being omitted for conciseness), the optimal incentives e^* and s^* are the solution to the following problem:

$$\min_{e, s \geq 0} [C(e, s, 0)]^+, \quad (16a)$$

$$\text{s.t. } D_m(e, s, 0) - D_m^0 \geq \beta D_m^0, \quad (16b)$$

$$\text{Mechanism [D]: } W(e, s, 0) \geq W^0, \quad (16c)$$

$$\Pi_s(e, s, 0) \geq \Pi_s^0. \quad (16d)$$

3.1. Direct Mechanism: Optimal Incentive (e^*, s^*)

By solving Problem (16), we obtain the optimal incentive (e^*, s^*) under mechanism [D].

Proposition 1. *The optimal congestion fee e^* and the optimal subsidy s^* under mechanism [D] are given by:*

$$e^* = \frac{\beta(\beta + 2)((d - p) - (r - d)x)^2}{2(\delta - 1)(x + 1)},$$

$$s^* = \frac{\beta((d - p) - (r - d)x)(2(\delta - 1)(x + 1) - (\beta + 2)((d - p) - (r - d)x))}{2(\delta - 1)(x + 1)}. \quad (17)$$

Further,

- The optimal congestion fee e^* strictly increases in β and strictly decreases in x .
- The optimal subsidy s^* strictly increases in β . If δ is large, s^* strictly decreases in x . However, if δ is small, there exists \tilde{x} such that s^* increases on $x < \tilde{x}$ and decreases on $x \geq \tilde{x}$.

Statements (i) and (ii) of Proposition 1 imply that, as the mixed-mode adoption target β becomes more aggressive, the commuter subsidy s^* and congestion fee e^* also increase to meet the adoption target, as expected. Additionally, as the last-mile distance x increases, the demand for driving also increases, and, hence, the congestion fee e^* decreases to maintain commuter welfare.

In contrast to the congestion fee e^* , the behavior of the optimal subsidy s^* is not monotonic in the last-mile distance x . To see why, let us consider the impact on mechanism [D] when x increases. As x increases, the mixed-mode adoption (before any government intervention)

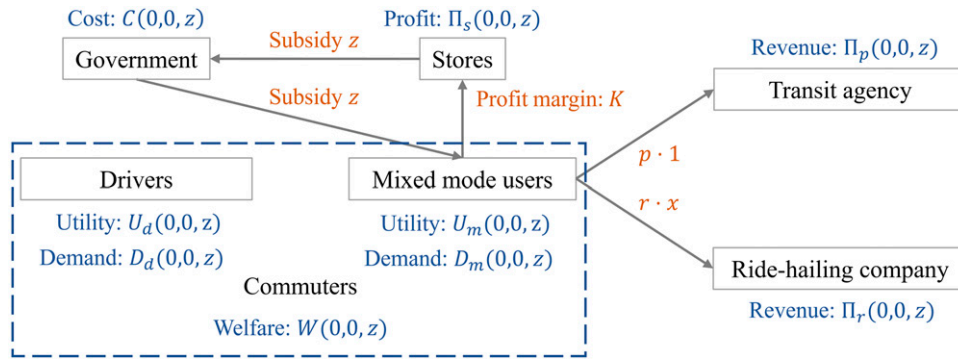
D_m^0 (as stated in (3)) decreases, which generates two competing effects. First, note that the transit-ridership constraint can be rewritten as $D_m(e, s, 0) \geq (\beta + 1)D_m^0$. Therefore, as D_m^0 decreases, the requirement for the mixed-mode commuters $D_m(e, s, 0)$ becomes less stringent. Consequently, the government can afford to reduce s , which we refer to as the “adoption effect.” Second, as D_m^0 decreases, commuter welfare decreases, due to fewer commuters receiving the subsidy. To ensure commuters are not worse off, the government needs to increase s , which we refer to as the “welfare effect.” Hence, whether the optimal subsidy s^* increases or decreases in x depends on which of these two effects dominate.

To examine when one effect dominates the other, first, consider the case when commuters strongly prefer driving over transit and ride-hailing (i.e., when δ is large). In this case, the welfare effect is weak (because the mixed-mode adoption D_m^0 is already low before any intervention), and the adoption effect dominates—leading s^* to decrease in x . However, when commuters are relatively indifferent between the two modes (i.e., when δ is small), the mixed-mode adoption D_m^0 is highly sensitive to small changes in x . As a result, as x increases, demand for the mixed mode drops sharply, which makes the welfare effect dominate—leading s^* to increase in x .¹⁸

Corollary 1. *There exists a threshold \hat{x} such that $e^* \geq s^*$ in mechanism [D] if and only if $x \leq \hat{x}$. Further, there exists a threshold $\bar{\delta} > 1$ such that $s^* > e^*$ in mechanism [D] for all $x \in (\underline{x}, \bar{x})$ if and only if $\delta \geq \bar{\delta}$.*

Corollary 1 states that for any adoption target β , the optimal congestion fee e^* is comparatively larger than the optimal subsidy s^* when the last-mile distance x is small. To see why, observe that when x is small, the mixed mode is already attractive to commuters, which makes providing a subsidy (per commuter) relatively costly; in this setting, the congestion fee e is the preferred lever for promoting the adoption of the mixed mode, due to its cost-efficiency. Conversely, when the last-mile distance x is large, more aggressive subsidies are required to meet the mixed-mode adoption target. Corollary 1 also implies that if

Figure 3. (Color online) Strategic Interactions Among Stakeholders Under the Indirect Mechanism [I]



δ is large, then $\hat{x} < \underline{x}$, which yields $s^* > e^*$ for all $x \in (\underline{x}, \bar{x})$. This occurs because when δ is large, most commuters prefer to drive, and so it is suboptimal for the government to set a high congestion fee, due to its degradation of commuter welfare.

Next, we examine whether the government can attain budget-neutrality (or positive revenue) by using congestion fees to offset commuter subsidies under mechanism [D].

Corollary 2. *It is impossible for mechanism [D] to be budget-neutral—that is, $C(e^*, s^*, 0) > 0$. Further, the minimal operating cost $C(e^*, s^*, 0)$ is higher when β is large or when x is small.*

The intuition behind Corollary 2 is as follows. For any budget-neutral incentive (e, s) (i.e., where the congestion fee e is selected to fully cover the cost of the subsidy s), it can be shown that the utility loss to drivers exceeds the utility gain to transit riders. In other words, for any subsidy s , there is no congestion fee e that can achieve budget-neutrality without a net decrease in total commuter welfare. As a result, the requirement that commuter welfare be nondecreasing in mechanism [D] can only be satisfied through a positive operating cost. Next, to see why $C(e^*, s^*, 0)$ is higher when β is large or when x is small, observe that, in this case, the mixed-mode adoption target βD_m^0 on the right-hand side of Constraint (16(b)) is large, which requires the mixed-mode adoption $D_m(e, s, 0)$ to also be large. Therefore, as more mixed-mode commuters receive the subsidy, mechanism [D] incurs a greater cost to the government. We can now summarize our main finding from mechanism [D] as follows.

Remark 1. The direct mechanism [D] can enable the government to meet the public transit adoption target β . However, this mechanism is costly: Attaining budget-neutrality is impossible without undermining commuter welfare.

Note that our commuter-utility function implies that commuters are equally sensitive to the subsidy and congestion fee; that is, $|\partial U_m(e, s, 0)/\partial s| = |\partial U_d(e, s, 0)/\partial e|$. Note that this symmetry may not hold if commuters are loss-averse. However, we find that the structure of our

main results continues to hold under loss aversion as well. Intuitively, if commuters perceive losses from congestion fees more strongly than gains from subsidies, then the government would need to *increase* the subsidy and *decrease* the congestion fee to satisfy the nondecreasing commuter welfare constraint, resulting in a higher total operating cost.¹⁹

Remark 1 naturally raises the following question: Does there exist an alternate mechanism that can enable the government to attain budget-neutrality, without negatively impacting commuter welfare? In the next section, we address this question by examining an alternative incentive program under which the subsidy is funded by a private enterprise, instead of congestion fees.

4. Indirect Mechanism: Funding Subsidies Through a Private Enterprise

Because most municipalities are financially constrained, self-funded or budget-neutral projects are preferred. In this section, we present the *indirect* mechanism [I], where the local government secures funding for the subsidy entirely from a private enterprise²⁰ and does not charge congestion fees. Note that the indirect mechanism is budget-neutral by design. We will therefore focus on the performance of mechanism [I] on the remaining evaluation criteria.

By using the same approach as in Section 3, we now present our model associated with mechanism [I], which corresponds to the special case of the general model framework presented in Section 2.2 when we set $(e, s) = (0, 0)$. In other words, by replacing (e, s, z) in the general formulation with $(0, 0, z)$, we can determine the performance metrics under mechanism [I]. The interactions among all stakeholders are depicted in Figure 3.

Because the viability of mechanism [I] depends on participation from the private enterprise, we assume the private subsidy z is set to maximize the enterprise's total profit. Based on the evaluation criteria (I)–(VI), the government's incentive-design problem associated with subsidy z under mechanism [I] can be

formulated as the following optimization problem:

$$\min_{z \geq 0} [C(0, 0, z)]^+, \quad (18a)$$

$$\text{s.t. } D_m(0, 0, z) - D_m^0 \geq \beta D_m^0, \quad (18b)$$

$$\text{Mechanism [I]: } W(0, 0, z) \geq W^0, \quad (18c)$$

$$\prod_s(0, 0, z) \geq \prod_s^0, \quad (18d)$$

$$z = \arg \max_z \prod_s(0, 0, z). \quad (18e)$$

Note that $C(0, 0, z) = 0$ for all z , by the definition of $C(e, s, z)$ as given in (11). Therefore, under mechanism [I], the government attains budget-neutrality as long as Problem (18) is feasible (in other words, mechanism [I] is budget-neutral by design). Note also that Formulation (18) may be infeasible if β is large, for the following reason. If β is large, then the private enterprise must issue a large subsidy to reach the adoption target. However, if the cost of providing a large subsidy exceeds the extra revenue earned by the enterprise (due to increased demand), then the participation constraint (18d) may be violated. Formally, let $\tau(x) = \min\left\{\frac{K}{((d-p)-x(r-d))} - 1, \bar{\tau}(x)\right\}$, where $\bar{\tau}(x) = \frac{(\delta-1)(x+1)}{(d-p)-(r-d)x} - 1$, as defined in Section 2.2. Note that $0 < \tau(x) \leq \bar{\tau}(x)$.

4.1. Indirect Mechanism: Optimal Subsidy z^*

By solving Problem (18), we obtain the optimal subsidy z^* in the following proposition.

Proposition 2. *Mechanism [I] is feasible if and only if $\beta \leq \tau(x)$. If $\beta \leq \tau(x)$, then the optimal subsidy is given by:*

$$z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\}, \quad (19)$$

where $\underline{z} = \beta((d-p) - x(r-d))$, $\hat{z} = \frac{1}{2}(K - ((d-p) - x(r-d)))$ and $\bar{z} = \min\{K, (\delta-1)(x+1)\} - ((d-p) - x(r-d))$. The minimal cost is zero, $C(0, 0, z^*) = 0$. Further,

- i. If the enterprise's profit margin K is high, the optimal subsidy z^* strictly increases in x .
- ii. If the enterprise's profit margin K is low, there exists \tilde{x} such that the optimal subsidy z^* strictly decreases on $x < \tilde{x}$ and strictly increases on $x \geq \tilde{x}$.

The intuition behind Proposition 2 is similar to that of Proposition 1. To elaborate, note that as the last-mile distance x increases, the mixed-mode adoption (before any government intervention) D_m^0 decreases, which generates two competing effects. First, the adoption effect (as described in Section 3.1) continues to play a role here, meaning the requirement for the corresponding mixed-mode commuters $D_m(0, 0, z)$ becomes less stringent as D_m^0 decreases. Consequently, the private enterprise can afford to reduce the subsidy z . Second, as D_m^0 decreases, the private enterprise has incentive to increase z , so that it can recoup the subsidy cost by increasing demand (i.e., a higher $D_m(0, 0, z)$). We shall refer to the latter effect as the “customer effect.” Note that the customer effect plays a role in

mechanism [I]—because the subsidy z is paid by the private enterprise—but is absent in mechanism [D]. Hence, whether the optimal subsidy z^* increases or decreases in x depends on which effect dominates.

To examine which effect dominates, let us consider the case where the enterprise's profit margin K is high. In this case, the enterprise's profit is highly sensitive to changes in D_m^0 . Therefore, the private enterprise has a stronger incentive to offer a higher subsidy to boost the customer base $D_m(0, 0, z)$, which makes the customer effect dominate the adoption effect. As a result, the subsidy z^* increases in x when K is large. The intuition behind the decrease in z^* when K is small is similar, which we omit for conciseness.

4.2. Comparison: Direct Mechanism [D] vs. Indirect Mechanism [I]

By considering Proposition 1 and Proposition 2 presented in Sections 3.1 and 4.1, we can compare the performance of the direct and indirect mechanisms, according to each of the six metrics discussed in Section 2.2. For ease of reference, we shall use superscript D and I to denote quantities obtained at optimal solutions in mechanisms [D] and [I], respectively. Also, in preparation, let $\tilde{\tau}(x) = \frac{\hat{z}^2 - (z^* - \hat{z})^2}{K((d-p) - (r-d)x)}$. The following result compares the performance of the two mechanisms, assuming $\beta \leq \tau(x)$.

Theorem 1. *Mechanism [I] outperforms mechanism [D] in terms of operating cost ($C^I = 0 < C^D$), mixed-mode adoption ($D_m^I \geq D_m^D$), commuter welfare ($W^I > W^D$), ride-hailing platform profit ($\Pi_r^I \geq \Pi_r^D$), and transit-agency revenue ($\Pi_p^I \geq \Pi_p^D$). Further,*

- i. If $\beta \leq \tilde{\tau}(x)$, mechanism [I] outperforms mechanism [D] on enterprise profit ($\Pi_s^I \geq \Pi_s^D$);
- ii. If $\beta \in (\tilde{\tau}(x), \tau(x)]$, mechanism [D] outperforms mechanism [I] on enterprise profit ($\Pi_s^I < \Pi_s^D$).

Theorem 1 states that the indirect mechanism [I] dominates the direct mechanism [D] in all performance metrics when the adoption target is not too large, $\beta \leq \tilde{\tau}(x)$. However, in the intermediate case, where $\tilde{\tau}(x) < \beta \leq \tau(x)$, the direct mechanism [D] will enable the private enterprise to generate a higher profit.

Recall that the main difference between mechanisms [I] and [D] is the funding source for the commuter subsidy. In mechanism [D], the government funds the subsidy by imposing congestion fees on drivers; in mechanism [I], the government simply coordinates the funding from the private-sector partner and is otherwise passive and budget-neutral. Theorem 1 highlights the benefits of a public-private partnership over congestion fees: In addition to achieving budget-neutrality, the government can both improve commuter welfare (by avoiding congestion fees on drivers) and support the private-sector partner (by boosting demand). As a result, all

stakeholders are left better off under mechanism [I] compared with [D], assuming the adoption target β is not too large. Moreover, note that mechanism [I] can dominate mechanism [D], despite containing only a single lever (the subsidy z). This result highlights that the gains from mechanism [I] are due to the coordination with the private enterprise.

Also, recall from Proposition 2 that when the adoption target is high $\beta > \tau(x)$, Formulation (18) is infeasible, meaning that there is no subsidy level z that can satisfy all of the criteria (I)–(VI) in Section 2.2. This observation enables us to make the following summarizing remark:

Remark 2. If the public transit adoption target is aggressive, $\beta > \tau(x)$, then the government should adopt mechanism [D] over mechanism [I]. If the adoption target is conservative, $\beta \leq \tau(x)$, the government should adopt mechanism [I] over mechanism [D] (even though the private enterprise can earn higher profit under mechanism [D] when β satisfies $\tilde{\tau}(x) < \beta \leq \tau(x)$).

The above remark has the following implications. When the municipal government has a very tight or no budget, the government should set a conservative adoption target $\beta \leq \tau(x)$. In this case, the government should adopt the indirect mechanism [I], which leaves all parties better off and, critically, does not require increased spending by the government. However, if the local government has ample funding, then it is feasible to set an aggressive transit-adoption target level $\beta \geq \tau(x)$, in which case the government should adopt and fund mechanism [D].

5. Alternative Formulation: Maximizing Commuter Welfare Under Budget-Neutrality

In Sections 3 and 4, we analyze the optimal incentive schemes under mechanisms [D] and [I] by minimizing operating cost $C(e, s, z)$. In this section, we analyze mechanisms [D] and [I] by considering an alternative objective: maximizing commuter welfare while maintaining budget-neutrality.

5.1. Alternative Direct Mechanism [D-A]

Recall from Remark 1 in Section 3.1 that it is not possible to implement the direct mechanism [D] in a budget-neutral manner without undermining commuter welfare. However, when a financially constrained municipality finds it too costly to implement mechanism [D], some loss in commuter welfare may be tolerated (especially if there is public support for policies that lower carbon emissions and traffic congestion). This observation motivates us to modify the original incentive-design problem (16) by maximizing commuter welfare subject to a budget-neutrality constraint (20(c)). Specifically, the modified problem

can be formulated as:

$$\max_{e, s \geq 0} (W(e, s, 0) - W^0), \quad (20a)$$

$$\text{s.t. } D_m(e, s, 0) - D_m^0 \geq \beta D_m^0, \quad (20b)$$

$$\text{Mechanism [D-A]: } C(e, s, 0) = 0, \quad (20c)$$

$$\Pi_s(e, s, 0) \geq \Pi_s^0, \quad (20d)$$

where D_m^0 , W^0 , and Π_s^0 are the demand for the mixed mode, commuter welfare, and profit of the private enterprise in the base model as defined in (3), (4), and (7), respectively. Analogous to Proposition 1, the next result characterizes the optimal incentive scheme under mechanism [D-A].

Proposition 3. *The optimal congestion fee e^* and the optimal subsidy s^* under the alternative direct mechanism [D-A] are given by:*

$$e^* = \frac{\beta(\beta + 1)((d - p) - (r - d)x)^2}{(\delta - 1)(x + 1)}$$

$$s^* = \frac{\beta((d - p) - (r - d)x)((\delta - 1)(x + 1) - (\beta + 1)((d - p) - (r - d)x))}{(\delta - 1)(x + 1)}$$

Further,

- i. The optimal congestion fee e^* increases in β and decreases in x .
- ii. If δ is large, s^* strictly decreases in x . However, if δ is small, there exists \tilde{x} such that s^* increases on $x < \tilde{x}$ and decreases on $x \geq \tilde{x}$.
- iii. There exists $\tilde{\beta}$ such that s^* increases on $\beta < \tilde{\beta}$ and decreases on $\beta \geq \tilde{\beta}$.

The above results resemble Proposition 1, except that the optimal subsidy s^* under the alternate mechanism [D-A] is no longer always increasing in β . Instead, the optimal subsidy s^* decreases β when the adoption target β is large. This difference is driven by the budget-neutrality constraint (20(c)). To elaborate, consider the case when β is large. In this case, the right-hand side of (20(b)) is large, which requires the mixed-mode demand $D_m(e, s, 0)$ to be large. As more mixed-mode commuters collect the subsidy, the government must reduce the subsidy s in order to maintain budget-neutrality, which leads s^* to decrease in the adoption target β .

Next, similar to Corollary 1, we compare the relative size of the optimal congestion fee and subsidy.

Corollary 3. *There exists \hat{x} such that $e^* \geq s^*$ in mechanism [D-A] if and only if $x \leq \hat{x}$. Further, there exists $\bar{\delta} > 1$ such that $s^* > e^*$ in mechanism [D-A] for all $x \in (\underline{x}, \bar{x})$ if and only if $\delta \geq \bar{\delta}$.*

Corollary 3 is analogous to Corollary 1: For any β , the government should set a higher congestion fee and a lower subsidy when the last-mile distance x is small, and vice versa when x is large.

Recall from Corollary 2 in Section 3.1 that the direct mechanism [D] is costly (i.e., $C(e^*, s^*, 0) > 0$) due to the commuter-welfare constraint (16(c)). Next, we present a counterpart to Corollary 2: To maintain budget-neutrality, commuter-welfare degradation is unavoidable under the alternative direct mechanism [D-A].

Corollary 4. *The alternative direct mechanism [D-A] will always result in commuter-welfare degradation; that is, $W(e^*, s^*, 0) < W^0$. Further, the reduction in commuter welfare, $W^0 - W(e^*, s^*, 0)$, under mechanism [D-A] is higher when the adoption target β is large and when the last-mile distance x is small.*

Corollary 4 reveals that mechanism [D-A] degrades commuter welfare more severely when the adoption target β is large. To elaborate, consider the case when the adoption target β is large. In this case, the corresponding mixed-mode demand $D_m(e, s, 0)$ has to be large in order to meet the adoption target. As more mixed-mode commuters collect the subsidy, the government must reduce the subsidy in order to maintain budget-neutrality, which causes the commuter welfare to deteriorate. Next, we consider why commuter-welfare reduction is higher when x is small. In this case, the mixed-mode adoption (before any government intervention) D_m^0 as stated in (3) is large, which makes the requirement for the mixed-mode adoption $D_m(e, s, 0)$ also large. Similar to the large- β case, as more mixed-mode commuters receive the subsidy, the government must increase the congestion fee e in order to maintain budget-neutrality, which reduces commuter welfare.

5.2. Alternative Indirect Mechanism [I-A]

In the same spirit as Section 5.1, we now consider a variation of mechanism [I], which we shall refer to as mechanism [I-A], where the objective is to maximize commuter welfare while maintaining budget-neutrality. To do so, we modify our original Program (18) presented in Section 4 as follows:

$$\max_{z \geq 0} (W(0, 0, z) - W^0), \quad (21a)$$

$$\text{s.t. } D_m(0, 0, z) - D_m^0 \geq \beta D_m^0, \quad (21b)$$

$$\text{Mechanism [I-A]: } C(0, 0, z) = 0, \quad (21c)$$

$$\Pi_s(0, 0, z) \geq \Pi_s^0, \quad (21d)$$

$$z = \arg \max_z \Pi_s(0, 0, z). \quad (21e)$$

In parallel with Proposition 2, the following proposition characterizes the optimal subsidy under the alternative indirect mechanism [I-A].

Proposition 4. *The alternative indirect mechanism [I-A] is feasible if and only if $\beta \leq \tau(x)$. When $\beta \leq \tau(x)$, the optimal subsidy is given by*

$$z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\},$$

where $\tau(x)$ is defined in Section 4 and $\underline{z}, \hat{z}, \bar{z}$ are defined in Proposition 2. The maximal welfare increment is strictly positive so that $W(0, 0, z^*) > W^0$.

Note the optimal subsidy z^* under mechanism [I-A] is equal to that under mechanism [I], as presented in Proposition 2. This is because Constraints (21(b)), (21(d)), and (21(e)), which are the same as Constraints (18(b)), (18(d)), and (18(e)), uniquely determine feasible private subsidy z^* and $C(0, 0, z^*) = 0$ by definition in (11). Thus, z^* is feasible and, thus, optimal to both Problems (18) and (21). Also, unlike mechanism [D-A] with deteriorating commuter welfare as stated in Corollary 4, Proposition 4 exerts that, when $\beta \leq \tau(x)$, mechanism [I-A] will always enhance commuter welfare.

5.3. Comparison: Two Alternative Mechanisms [D-A] and [I-A]

We now compare the performance of the two alternative mechanisms, [D-A] and [I-A]. Using the same approach as presented in Section 4.2, we can retrieve the relevant performance metrics from (10)–(15) by substituting (e, s, z) with $(e^*, s^*, 0)$ and $(0, 0, z^*)$.

Theorem 2. *The alternative indirect mechanism [I-A] and the alternative direct mechanism [D-A] both attain budget-neutrality ($C^I = C^D = 0$). However, alternative indirect mechanism [I-A] outperforms alternative direct mechanism [D-A] in terms of mixed-mode adoption ($D_m^I \geq D_m^D$), commuter welfare ($W^I > W^0 > W^D$), ride-hailing platform profit ($\Pi_r^I \geq \Pi_r^D$), and transit-agency revenue ($\Pi_p^I \geq \Pi_p^D$). Further,*

1. *If $\beta \leq \tilde{\tau}(x)$, mechanism [I-A] outperforms mechanism [D-A] in terms of enterprise profit ($\Pi_s^I \geq \Pi_s^D$),*

2. *If $\beta \in (\tilde{\tau}(x), \tau(x))$, mechanism [D-A] outperforms mechanism [I-A] in terms of enterprise profit ($\Pi_s^I < \Pi_s^D$), where $\tau(x)$ and $\tilde{\tau}(x)$ are defined in Sections 4 and Section 2.2, respectively.*

Using the above proposition, we summarize our observations in the following remark.

Remark 3. *Suppose, in addition to budget-neutrality, the government prioritizes commuter welfare. Then, the government should adopt the alternative direct mechanism [D-A] when the adoption target satisfies $\beta > \tau(x)$, and adopt the alternative indirect mechanism [I-A] when $\beta \leq \tau(x)$.*

Remark 2 and Remark 3 imply that the dominance of the indirect mechanism [I] is robust, regardless of whether the goal is to maximize commuter welfare or to minimize operating cost. Therefore, when the municipal government is financially constrained, it is advisable for the government to set a conservative transit-adoption target $\beta \leq \tau(x)$, and adopt the indirect mechanism [I]. However, if the government has sufficient funding, or if the commuters are willing to

accept lower welfare (e.g., in support of a reduction in carbon emissions and traffic congestion), then the government can afford to set an aggressive adoption target $\beta > \tau(x)$ and adopt the direct mechanism [D].

6. Conclusion

Motivated by the increased focus on public transit and urban mobility by municipal governments, we have analyzed two mechanisms for improving public transit ridership. Both mechanisms are intended to address the last-mile gap by providing subsidies to commuters who adopt a mixed mode of transportation that combines ride-hailing with public transit. The main differences between the two mechanisms are the role of the municipal government and the source of funding for the subsidy. In the direct mechanism [D], the government provides a ride-hailing subsidy to commuters that adopt the mixed mode and charges a congestion fee to commuters who travel by personal vehicle. The congestion fee is used to offset the cost of the subsidy and also serves as an additional incentive for transit adoption (by making driving more costly). In the indirect mechanism [I], the government partners with a private enterprise that provides the funding for the subsidy and does not charge a congestion fee. We present analytical results that characterize the optimal incentives under these two mechanisms, as well as the impact on all involved stakeholders.

Our findings offer several prescriptions for policy makers who are interested in increasing public transit ridership through partnerships with the private sector. First, in the direct mechanism, it's advisable to set a large congestion fee and a small subsidy when public transit coverage is high (i.e., when the last-mile distance is small), and, conversely, when the public transit coverage is low, the optimal incentive requires more aggressive subsidies. This result is driven by the opposing effects that subsidies and congestion fees have on commuter welfare and operating cost. We also find that the dependence of the optimal subsidy on the transit-coverage level depends on the relative preference between driving and taking public transit. In particular, we find that when commuters strongly prefer driving, jurisdictions with larger last-mile distances should set lower subsidies, but this behavior can be reversed if commuters only slightly prefer driving over public transit.

Second, we find that the government cannot fully recover the cost of providing subsidies by collecting congestion fees. Specifically, we show that the direct mechanism cannot be budget-neutral unless the government is willing to accept a decrease in commuter welfare. This suggests that attempting to implement commuter subsidies and congestion fees in a budget-neutral manner may be ill-advised if the government is sensitive to commuter welfare. However, in the event that the government can obtain subsidy funding from a private

enterprise (who benefits from increased foot traffic at the transit station or increased views of an advertisement on a mobile app) in lieu of charging congestion fees (as in the indirect mechanism), we show that public transit ridership can be increased without degrading commuter welfare. However, because the implementation of an indirect mechanism requires the participation of the private enterprise, we find that this indirect mechanism is only viable if the mixed-mode adoption target is modest. In other words, only the direct mechanism can enable the government to achieve an ambitious adoption target, and the implementation of the direct mechanism is not budget-neutral: It will require extra funding.

Third, although the government has one less "lever" in the indirect mechanism [I] (due to the absence of the congestion fee), it can dominate the direct mechanism [D] if the adoption target is modest. Specifically, when the adoption target is modest, the indirect mechanism can benefit all stakeholders: the government, commuters, the public transit agency, the ride-hailing platform, and the private enterprise. This finding suggests that, for jurisdictions that wish to increase transit ridership, but are severely budget-constrained, it may be more fruitful to fund ride-hailing subsidies through partnerships with the private sector than by charging congestion fees. Moreover, this dominance result is robust to an alternate specification of mechanism [I] (namely, when the objective is to maximize commuter welfare instead of enterprise profit).

As an initial attempt to explore different incentive mechanisms for improving public transit ridership, our model has several limitations that we wish to highlight. First, for tractability, we restrict attention to two modes of travel—driving and the mixed mode. Although other means of commuting are less common (such as commuting by ride-hailing), including them as travel modes may affect our results. Second, subsidies for ride-hailing trips may cause commuters to switch from other last-mile travel modes, such as walking, biking, or busing, which may have consequences not discussed in this paper. Further, to focus on the economic viability of different incentive mechanisms, we have abstracted away from operational issues, such as capacity, traffic congestion, and the spatial nature of public transit infrastructure. Considering these features may yield additional insight into the design of incentive mechanisms for improving transit ridership. We shall defer these issues to future research.

We conclude by offering several potential directions for future work. First, it may be fruitful to examine other forms of partnership between the transit agency, ride-hailing platform, and the private enterprise. For example, because the ride-hailing platform benefits from the commuter subsidy (due to increased demand), it may be worthwhile to examine partnerships where the cost of the subsidy is shared between the municipal

government and the ride-hailing platform. Second, our work is relevant to other settings where two firms with “complementary capacity” engage in a mutually beneficial partnership. For example, FedEx offers a “SmartPost” delivery service, in which FedEx maintains responsibility for the long-haul transportation of goods, while the U.S. Postal Service (USPS) handles the last-mile delivery between a USPS center and a customer’s home. Our work can serve as a springboard for analyzing partnerships of this nature. Third, we have modeled commuter welfare at an aggregate level; it may also be worthwhile to investigate mechanisms that can improve welfare for *all* commuters (e.g., by reducing traffic congestion for drivers). Lastly, budget-neutral incentive mechanisms have received relatively little attention in the operations literature, but may be valuable in other settings where government funds are limited. Our findings suggest that an appropriately designed public-private partnership may be a cost-effective solution for governments that wish to improve adoption of other socially beneficial goods or services (e.g., electric vehicles).

Endnotes

¹ Single-occupancy vehicles emit 1.5 times the CO₂ emissions of buses and four times the emissions of subways (Hodges and Martin 2010). With respect to traffic, public transit has been found to be critical in reducing congestion during peak hours (Anderson 2014).

² The notion of delivering public services in a budget-neutral manner has recently been proposed by the Centers for Medicare and Medicaid Services; the reader is referred to CMS (2018) for details.

³ Information technology has been successfully leveraged to help commuters plan and pay for multimodal trips within a mobile app; examples include Whim in Helsinki, Citymapper in London, Moovel in Germany, UbiGo in Gothenburg, TAP in Los Angeles, Clipper in San Francisco, and Opal in Sydney (Cole 2017, Goodall et al. 2017).

⁴ There is also evidence to suggest that ride-hailing may be contributing to the decline in public transit ridership and adding to traffic congestion in city centers (Brown 2020). Partnerships between public transit agencies and private ride-hailing platforms may facilitate cooperation instead of competition, which can be beneficial to both parties (i.e., increased transit ridership in the city center and increased demand for ride-hailing in suburban areas).

⁵ In the context of public transit, “coverage” broadly refers to the accessibility of transit stations by the general population (i.e., commuters, in our context) (Tomer et al. 2011).

⁶ Congestion fees have been adopted by many municipal governments, including Singapore, Hong Kong, London, Milan, and Stockholm. Singapore was the first to introduce congestion pricing as a tool to control traffic volume, where citizens pay for fees when they enter city-center areas (Development Asia 2018). London introduced congestion fees in 2003, where there is a charge for entering London’s congestion charging zone (13 square miles) between 7 a.m. and 6 p.m. on weekdays (Transport for London 2019). Since the introduction of congestion fees, traffic congestion and private vehicle usage in London has dropped by 25% and 39%, respectively (Badstuber 2018). Similarly, in Milan, public ridership increased 12.5% from 2007 to 2013 (Crocchi 2016).

⁷ An expanded version of the paper that considers extensions to our model can be found online at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3540566. Our structural results continue to hold in those settings as well.

⁸ We focus on suburban commuters for whom walking to the transit station is prohibitively costly. Accordingly, we exclude the mixed mode that combines walking and public transit. Similarly, we exclude the case where commuters combine electric scooters or bicycles and public transit because (a) commuters’ comfortable biking distance is limited (Rastogi and Rao 2003); and (b) less than 4% of commuters use scooters or bicycles to get to/from transit stations (Federal Highway Administration 2017). We also exclude the mixed mode that combines driving and public transit, due to limited parking spaces near transit stations in the United States. Commuters who exclusively use ride-hailing to commute are also not considered here (only 0.2% of suburban residents commute via taxi or ride-hailing services (Federal Highway Administration 2017)).

⁹ Empirical evidence suggests that a commuter’s willingness to pay for transportation is increasing in the travel distance (Van Ommeren et al. 2000, Jou et al. 2012). For tractability, we assume a linear relationship between willingness to pay and distance, given by the parameter V .

¹⁰ Recent survey data have shown that commuters generally prefer driving to public transit and ride-hailing services (Zhu and Fan 2018). Our main results also persist in a model where commuters prefer ride-hailing to public transit, so that the additional value associated with ride-hailing is $l \cdot V$, where $\delta > \ell \geq 1$. For ease of exposition, we assume $\ell = 1$ throughout.

¹¹ Commuter valuation for each travel mode can vary based on demographic features such as age, income level, or health conditions (Zhu and Fan 2018). For simplicity, we assume the valuation $V \sim U[0, 1]$.

¹² The cost of driving is estimated to be \$0.76/mile, assuming a mileage of 10,000 miles per year (American Automobile Association 2018). Ride-hailing is estimated to cost more than \$1.07/mile (Lyft 2021). The cost of commuting by public transit is typically much lower; for example, the cost of public transit in Los Angeles is estimated to be \$0.2/mile, assuming an average commute distance of 16 miles (Leonard 2019).

¹³ The average vehicle-occupancy rate of public transit in 2017 was less than 30% (APTA 2019d). Ride-hailing services also have ample capacity: The average idle rate of Uber drivers is 30%–40% (Currie 2018, Brown 2020).

¹⁴ Note that if $ac \geq 1$, then $(k - c)[1 - ka]^+ D_m^0 \leq 0$ for any $D_m^0 \geq 0$ and $k \geq 0$.

¹⁵ Observe from (1) that the utilities U_d^0 and U_m^0 are based on parameters x , d , r , and p . Hence, the lump-sum congestion fee e and subsidy s and z in (8) can also be expressed as a percentage fee and subsidy through an appropriate scaling of those parameters.

¹⁶ Because of an anticipated budget deficit through 2026, the U.S. Department of Transportation is likely to further reduce transfers to states and transit agencies (Kirk and Mallett 2020). Similarly, Transport for London faces a grant cut from the government of £700m a year (Edwards 2018).

¹⁷ Note that although transit agencies receive funding from local governments, they operate under separate budgets (Federal Transit Administration 2020). Therefore, we model the transit agency as a distinct stakeholder from the local government.

¹⁸ Note that even when δ is small, the welfare effect ceases to dominate the adoption target effect when the last-mile distance is large ($x \geq \bar{x}$), due to low demand for the mixed mode.

¹⁹ Similar to Homonoff (2018), we can extend our model to capture one aspect of loss aversion by defining the commuter utilities associated with the mixed mode and driving as $U_m(e, s, z) = U_m^0 + s + z$ and $U_d(e, s, z) = U_d^0 - \rho \cdot e$, respectively, where $\rho \geq 1$ captures the strength of the commuter’s aversion to the congestion fee. The evaluation criteria (I)–(VI) can be redefined accordingly. Using a parallel argument to the proof of Corollary 2, it is straightforward to verify that the optimal operating cost is strictly positive for all $\rho \geq 1$.

²⁰ As explained in Section 1, the transit agency can generate revenue from private companies, who post in-app video ads via mobile apps, and the indirect mechanism can be operationalized through a mobile app that allows the government to collect fares from riders and subsidies from the private sector.

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Partnerships in Urban Mobility: Incentive Mechanisms for Improving Public Transit Adoption

Online Appendix

Lemma 1. *In mechanism [D], the following inequality holds for all (e, s) such that $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$, or equivalently, $e + s \leq (\delta - 1)(x + 1) - ((d - p) - (r - d)x)$:*

$$[C(e, s, 0)]^+ \geq C(e, s, 0) \geq \frac{(\beta((d-p) - (r-d)x))^2}{2(\delta-1)(x+1)}.$$

Proof Note that when requiring $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$, constraint (16b) implies

$$D_m(e, s, 0) - D_m^0 = \frac{e + s}{(\delta - 1)(x + 1)} \geq \beta D_m^0 = \beta \frac{(d - p) - (r - d)x}{(\delta - 1)(x + 1)}.$$

Because $\delta > 1$ and $x > 0$, it follows that

$$e + s \geq \beta((d - p) - (r - d)x), \quad (1)$$

which holds with equality only if constraint (16b) is binding. Next, using (11), the operational cost is given by

$$C(e, s, 0) = sD_m(e, s, 0) - eD_d(e, s, 0) = \frac{e((d-p) - (r-d)x - (\delta-1)(x+1)) + s((d-p) - (r-d)x)}{(\delta-1)(x+1)} + \frac{(e+s)^2}{(\delta-1)(x+1)}.$$

Further, replacing (e, s, z) with $(e, s, 0)$ in (8) and (12), we obtain $U_d(e, s, 0)$, $U_m(e, s, 0)$ and $W(e, s, 0)$. Then the welfare *increment* is given by

$$W(e, s, 0) - W^0 = \left[\int_0^{v(e,s,0)} U_m(e, s, 0) dV + \int_{v(e,s,0)}^1 U_d(e, s, 0) dV \right] - \left[\int_0^{v^0} U_m^0 dV + \int_v^1 U_d^0 dV \right] \quad (2)$$

$$= \frac{e((d-p) - (r-d)x - (\delta-1)(x+1)) + s((d-p) - (r-d)x)}{(\delta-1)(x+1)} + \frac{(e+s)^2}{2(\delta-1)(x+1)}. \quad (3)$$

It follows that for all (e, s) such that $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$, it holds that

$$C(e, s, 0) = (W(e, s, 0) - W^0) + \frac{(e+s)^2}{2(\delta-1)(x+1)}. \quad (4)$$

Because constraint (16c) implies $W(e, s, 0) - W^0 \geq 0$, it follows from (4) that $C(e, s, 0) \geq \frac{(e+s)^2}{2(\delta-1)(x+1)}$, which holds with equality only when constraint (16c) is binding. Further, because $x < \bar{x}$, $(d-p) - (r-d)x > 0$. It follows from (1) that for all (e, s) ,

$$[C(e, s, 0)]^+ \geq C(e, s, 0) \geq \frac{(e+s)^2}{2(\delta-1)(x+1)} \geq \frac{(\beta((d-p) - (r-d)x))^2}{2(\delta-1)(x+1)}, \quad (5)$$

Because $\frac{(\beta((d-p)-(r-d)x))^2}{2(\delta-1)(x+1)} \geq 0$, it follows that (5) holds with equality if and only if both constraints (16b) and (16c) are binding. \square

Proof of Proposition 1. The proof proceeds in two steps. In Step 1, we find the optimal solution (e^*, s^*) . In Step 2, we prove statements (i) and (ii).

Step 1. To solve for (e^*, s^*) , we consider two cases based on the definition of $D_m(e, s, 0)$ in (10), which implies $D_m(e, s, 0) = \min \left\{ 1, \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)} \right\}$: where (e, s) satisfies $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$, and where (e, s) satisfies $D_m(e, s, 0) = 1$. Then, we show that the optimal cost in is lower in the first case.

Case 1: Suppose $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$. This condition is equivalent to

$$e + s \leq (\delta - 1)(x + 1) - ((d - p) - (r - d)x), \quad (6)$$

which we add to problem (16) as an additional constraint in this case. Because the objective is to minimize $[C(e, s, z)]^+$, to find an optimal solution it suffices to construct a feasible solution (e_1, s_1) that attains the lower bound in Lemma 1. Let (e_1, s_1) be given by the unique solution to the equations

$$\begin{aligned} D_m(e, s, 0) - D_m^0 &= \beta D_m^0, \\ W(e, s, 0) &= W^0, \end{aligned}$$

which correspond to constraints (16b) and (16c). Solving for (e, s) yields the following unique solution:

$$\begin{aligned} e_1 &= \frac{\beta(\beta + 2)((d - p) - (r - d)x)^2}{2(\delta - 1)(x + 1)} \\ s_1 &= \frac{\beta((d - p) - (r - d)x)(2(\delta - 1)(x + 1) - (\beta + 2)((d - p) - (r - d)x))}{2(\delta - 1)(x + 1)}. \end{aligned}$$

Next, we show that (e_1, s_1) is feasible. By construction, (e_1, s_1) satisfies constraints (16b) and (16c) with equality. Note that $e_1 + s_1 = \beta((d - p) - (r - d)x)$. Because $0 < \beta < \bar{\tau}(x)$, (e_1, s_1) satisfies constraint (6). Next, because $\delta > 1$ and $x > \underline{x}$, we have $e_1 > 0$. In addition, because $0 < \beta < \bar{\tau}(x)$, we also have $s_1 > 0$. Further, for any (e, s) , the revenue *increment* of nearby merchants is given by $\Pi_s(e, s, 0) - \Pi_s^0 = K \frac{e+s}{(\delta-1)(x+1)}$, which follows from (15). Because $e_1 > 0$ and $s_1 > 0$, it follows that $\Pi_s(e, s, 0) - \Pi_s^0 > 0$, and thus (e_1, s_1) satisfies constraint (16d). Hence, (e_1, s_1) is feasible to (16). Because formulation (16) minimizes $[C(e, s, 0)]^+$, and (e_1, s_1) achieves the lower bound of $[C(e, s, 0)]^+$ presented in Lemma 1, it follows that (e_1, s_1) is optimal to (16) when restricting (e, s) to satisfy $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$.

Case 2: Suppose $D_m(e, s, 0) = 1$. It follows that $D_d(e, s, 0) = 0$ and $v(e, s, 0) = 1$. Constraint (16c) can then be rewritten as

$$W(e, s, 0) - W^0 = \int_0^1 U_m(e, s, 0) dV - \left[\int_0^{v^0} U_m^0 dV + \int_{v^0}^1 U_d^0 dV \right] \quad (7)$$

$$= s - \frac{((\delta - 1)(x + 1) - ((d - p) - x(r - d)))^2}{2(\delta - 1)(x + 1)}. \quad (8)$$

Let $\phi = \frac{((\delta-1)(x+1)-((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)}$. Then constraint (16c) requires that (e, s) satisfies $s \geq \phi$. Using (11) and the fact that $D_m(e, s, 0) = 1$, we have $C(e, s, 0) = s \geq 0$. Therefore, $[C(e, s, 0)]^+ = C(e, s, 0) \geq 0$. Because $0 < \beta < \bar{\tau}(x)$, it is straightforward to verify that $\phi > C(e_1, s_1, 0) = [C(e_1, s_1, 0)]^+$. Hence, for all

(e, s) that satisfies $D_m(e, s, 0) = 1$, $[C(e, s, 0)]^+ > [C(e_1, s_1, 0)]^+$. Therefore, the optimal solution to (16), denoted by (e^*, s^*) , must satisfy $D_m(e^*, s^*, 0) = \frac{(d-p)-(r-d)x+e^*+s^*}{(\delta-1)(x+1)}$. It follows that $(e^*, s^*) = (e_1, s_1)$.

Step 2. Next, we prove statements (i) and (ii). (i). It is straightforward to verify that $\frac{\partial e^*}{\partial \beta} = \frac{(\beta+1)((d-p)-(r-d)x)^2}{(\delta-1)(x+1)} > 0$. Next, by taking the first derivative of e^* with respect to x , we have $\frac{\partial e^*}{\partial x} = \frac{\beta(\beta+2)((x+1)^2(r-d)^2-(r-p)^2)}{2(\delta-1)(x+1)^2}$. Because $x \in (\underline{x}, \bar{x})$ and $r > d > p > 0$, it follows that $(x+1)^2(r-d)^2 < (r-d+d-p)^2 = (r-p)^2$, and thus $\frac{\partial e^*}{\partial x} < 0$. (ii). Note that $\frac{\partial s^*}{\partial \beta} = \frac{((d-p)-(r-d)x)((\delta-1)(x+1)-(\beta+1)((d-p)-(r-d)x))}{(\delta-1)(x+1)}$. Because $\delta > 1$, $\beta < \bar{\tau}(x)$ and $x \in (\underline{x}, \bar{x})$, it follows that $\frac{\partial s^*}{\partial \beta} > 0$. To compute $\frac{\partial s^*}{\partial x}$, first note that $\frac{\partial^2 s^*}{\partial x^2} = -\frac{\beta(\beta+2)(r-p)^2}{(\delta-1)(x+1)^3} < 0$, meaning s^* is strictly concave in x and $\frac{\partial s^*}{\partial x}$ decreases in x . We next check the first derivatives at the boundaries, i.e. $\frac{\partial s^*}{\partial x}\Big|_{x=\bar{x}}$ and $\frac{\partial s^*}{\partial x}\Big|_{x=\underline{x}}$. Note $\frac{\partial s^*}{\partial x}\Big|_{x=\bar{x}} = -\beta(r-d) < 0$. If $\delta < \delta_1 = 1 + d - p$, then $\underline{x} = \frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}$, and thus $\frac{\partial s^*}{\partial x}\Big|_{x=\underline{x}} = \frac{1}{2}\beta((\beta+2)(\delta-1)+2(\beta+1)(r-d)) > 0$; if $\delta \geq \delta_1$, then $\underline{x} = 0$, and thus $\frac{\partial s^*}{\partial x}\Big|_{x=\underline{x}} = -\frac{\beta(2(\delta-1)(r-d)-(\beta+2)(d-p)(2r-d-p))}{2(\delta-1)}$. Similarly, if $\delta < \delta_2 = \frac{(\beta+2)(d-p)(2r-d-p)}{2(r-d)} + 1$, $\frac{\partial s^*}{\partial x}\Big|_{x=0} > 0$; if $\delta \geq \delta_2$, $\frac{\partial s^*}{\partial x}\Big|_{x=0} \leq 0$. Because $\delta_2 - \delta_1 = \frac{\beta(d-p)(2r-d-p)+2(d-p)(r-p)}{2(r-d)} > 0$, we have $\delta_2 > \delta_1$. Therefore, if $\delta \geq \delta_2$, $\frac{\partial s^*}{\partial x}\Big|_{x=\underline{x}} \leq 0$, and if $1 < \delta < \delta_2$, $\frac{\partial s^*}{\partial x}\Big|_{x=\underline{x}} > 0$. Therefore, by continuity of $\frac{\partial s^*}{\partial x}$, if $\delta \geq \delta_2$, then s^* strictly decreases in $x \in (\underline{x}, \bar{x})$. If $\delta < \delta_2$, then there exists $\tilde{x} \in (\underline{x}, \bar{x})$ such that s^* strictly increases in x on $(\underline{x}, \tilde{x})$ and strictly decreases in x on $[\tilde{x}, \bar{x})$. \square

Proof of Corollary 1. Using the expressions for (e^*, s^*) in Proposition 1, we have

$$e^* - s^* = \beta((d-p) - (r-d)x) \left(\frac{(\beta+2)((d-p) - (r-d)x)}{(\delta-1)(x+1)} - 1 \right).$$

Because $x \in (\underline{x}, \bar{x})$, $(d-p) - (r-d)x > 0$. Further, because $\delta > 1$, we have $e^* \geq s^*$ if and only if $x \leq \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)((\beta+2)(r-d)+\delta-1)}$. The result follows by setting $\hat{x} = \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)((\beta+2)(r-d)+\delta-1)}$. Next, note that because $\delta > 1$, $r > p$ and $r > d$, it follows that $\hat{x} < \bar{x}$. Define $\bar{\delta} = (\beta+2)(d-p) + 1$, which satisfies $\bar{\delta} > 1$. It remains to show that $\underline{x} \leq \hat{x}$ if and only if $\delta \geq \bar{\delta}$. From the definition of \underline{x} , if $\delta < 1 + d - p$, then $\underline{x} = \frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}$. It is straightforward to check that $\hat{x} \in (\underline{x}, \bar{x})$. If $\delta \geq 1 + d - p$, then $\underline{x} = 0$, and thus $\hat{x} \leq \underline{x}$ only if $\delta \geq \bar{\delta}$, where $\bar{\delta} > 1 + d - p$. Therefore, $s^* > e^*$ holds for all $x \in (\underline{x}, \bar{x})$ if and only if $\hat{x} \leq \underline{x}$, which holds if and only if $\delta \geq \bar{\delta}$.

Proof of Corollary 2. Using the expressions for e^* and s^* from Proposition 1 and the cost function given in (11), the total cost under (e^*, s^*) is $C(e^*, s^*, 0) = \frac{\beta^2((d-p)-(r-d)x)^2}{2(\delta-1)(x+1)}$. Because $x < \bar{x}$ and $\delta > 1$, $C(e^*, s^*, 0) > 0$. By noting that $\frac{\partial C(e^*, s^*, 0)}{\partial \beta} = \frac{\beta((d-p)-x(r-d))^2}{(\delta-1)(x+1)} > 0$, we conclude that $C(e^*, s^*, 0)$ increases in β . Next, note $\frac{\partial C(e^*, s^*, 0)}{\partial x} = \frac{\beta^2((x+1)^2(r-d)^2-(r-p)^2)}{2(\delta-1)(x+1)^2}$. Because $x \in (\underline{x}, \bar{x})$, we have $0 < (x+1)(r-d) < (r-p)$. It follows that $\frac{\partial C(e^*, s^*, 0)}{\partial x} < 0$. \square

Proof of Proposition 2. From the definition in (11), we have $[C(0, 0, z)]^+ = C(0, 0, z) = 0$. Thus, under mechanism [I], the operating cost is always 0. It follows that any feasible solution to problem (18) is optimal. Therefore, solving for the optimal subsidy z^* is equivalent to finding the optimal solution to the following

problem:

$$\max_{z \geq 0} \Pi_s(0, 0, z) \quad (9a)$$

$$\text{s.t. } D_m(0, 0, z) - D_m^0 \geq \beta D_m^0 \quad (9b)$$

$$W(0, 0, z) \geq W^0 \quad (9c)$$

$$\Pi_s(0, 0, z) \geq \Pi_s^0 \quad (9d)$$

The proof is composed of three steps. In Step 1, we prove (9) is feasible if and only if $\beta \leq \tau(x)$, which is also a sufficient and necessary condition for formulation (18) to be feasible. Next, in Step 2, we solve for the optimal solution z^* to (9), which is also optimal to (18). Finally, in Step 3, we conduct the comparative statics analysis of z^* .

Step 1. To construct the sufficient and necessary condition for problem (9) to be feasible, we establish the conditions under which the feasible solution set is non-empty. Let us first define the feasible set. Let $\tilde{z} = (\delta - 1)(x + 1) - ((d - p) - (r - d)x)$. To find all the feasible solutions to problem (9), we consider two cases based on the definition of $D_m(0, 0, z)$ in (10), which implies $D_m(0, 0, z) = \min\left\{1, \frac{(d-p)-(r-d)x+z}{(\delta-1)(x+1)}\right\}$: $D_m(0, 0, z) = \frac{(d-p)-(r-d)x+z}{(\delta-1)(x+1)}$ and $D_m(0, 0, z) = 1$. In preparation, note that because $\underline{x} < x < \bar{x}$ and $\delta > 1$, we have $(\delta - 1)(x + 1) > ((d - p) - x(r - d)) > 0$.

Case 1: Suppose $D_m(0, 0, z) = \frac{(d-p)-(r-d)x+z}{(\delta-1)(x+1)}$, which is equivalent to $z \leq \tilde{z}$. In this case, we first show that any $z \geq 0$ satisfies constraint (9c). Using the welfare expression given in (12), the commuter welfare increment is $W(0, 0, z) - W^0 = \frac{z(2(d-p)-2(r-d)x+z)}{2(\delta-1)(x+1)}$. Because $\delta > 1$ and $x \in (\underline{x}, \bar{x})$, we have $W(0, 0, z) - W^0 \geq 0$ if $z \geq 0$. Therefore, constraint (9c) is satisfied for all $z \geq 0$. Next, we show that when restricting $z \leq \tilde{z}$, the feasible region of problem (9) is $[\underline{z}, \bar{z}]$, where $\underline{z} = \beta((d - p) - x(r - d))$ and $\bar{z} = \min\{K, (\delta - 1)(x + 1)\} - ((d - p) - x(r - d))$. To do so, we prove that constraint (9b) is satisfied if and only if $z \geq \underline{z}$, and constraint (9d) is satisfied if and only if $0 \leq z \leq K - ((d - p) - x(r - d))$. First, using the definition of D_m^0 in (3), it is straightforward to verify that constraint (9b) holds if and only if $\frac{z}{(\delta-1)(x+1)} \geq \beta \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}$. Because $\delta > 1$, it follows that constraint (9b) holds if and only if $z \geq \underline{z} = \beta((d - p) - x(r - d))$. Because $x \leq \bar{x}$, it follows that $\underline{z} > 0$. Next, using the revenue expression in (15), when restricting $z \leq \tilde{z}$, it is straightforward to verify that constraint (9d) holds if and only if $0 \leq z \leq K - ((d - p) - x(r - d))$. Therefore, when adding the additional constraint $z \leq \tilde{z}$ to problem (9), the feasible region is $[\underline{z}, \min\{\tilde{z}, K - ((d - p) - x(r - d))\}]$, or equivalently, $[\underline{z}, \bar{z}]$.

Case 2: Suppose $D_m(0, 0, z) = 1$, which is equivalent to $z \geq \tilde{z}$. Let $\dot{z} = K(1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)})$. It is straightforward to verify that constraint (9d) is satisfied if and only if $z \leq \dot{z}$. Next, we show that constraints (9b) and (9c) are satisfied if $z \geq \tilde{z}$. First, note that

$$D_m(0, 0, z) - D_m^0 = 1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)} \geq \beta \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}$$

holds for $z \geq \tilde{z}$ because $\beta < \tau(x)$; thus constraint (9b) is satisfied for all $z \geq \tilde{z}$. Next, from the definition in

(10), if $z \geq \tilde{z}$, then

$$\begin{aligned} W(0, 0, z) - W^0 &= \frac{(\delta - 1)(x + 1)((x + 1)(2d - \delta) - 2p - 2rx + 2z + x + 1) - (-d(x + 1) + p + rx)^2}{2((\delta - 1)(x + 1))} \\ &> \frac{(\delta - 1)^2(x + 1)^2 - ((d - p) - x(r - d))^2}{2((\delta - 1)(x + 1))} \\ &\geq 0, \end{aligned}$$

which implies constraint (9c) is satisfied for all $z \geq \tilde{z}$. Therefore, constraint (9d) is satisfied if and only if $z \leq \hat{z}$, and constraints (9b) and (9c) are satisfied if $z \geq \tilde{z}$.

To summarize Case 1 and Case 2, note that $\bar{z} \leq \tilde{z}$, and thus the feasible region of problem (9) is $[\underline{z}, \bar{z}] \cup [\tilde{z}, \hat{z}]$. Now that we have constructed the feasible set, next we prove $[\underline{z}, \bar{z}] \cup [\tilde{z}, \hat{z}]$ is non-empty if and only if $\beta \leq \tau(x)$. The derivation consists of two cases: $K < (\delta - 1)(x + 1)$ and $K \geq (\delta - 1)(x + 1)$. If $K < (\delta - 1)(x + 1)$, it is straightforward to verify that $\hat{z} < \tilde{z}$, in which case the feasible region is $[\underline{z}, \bar{z}]$; note that this interval is non-empty if and only if $\underline{z} \leq \bar{z}$, or equivalently, $\beta \leq \frac{K}{((d-p)-(r-d)x)} - 1$. If $K \geq (\delta - 1)(x + 1)$, then $\bar{z} = \tilde{z} \leq \hat{z}$, in which case the feasible region is $[\underline{z}, \hat{z}]$; note that this interval is non-empty if and only if $\underline{z} \leq \hat{z}$, or equivalently, $\beta \leq \frac{K}{(\delta - 1)(x + 1)} \bar{\tau}(x)$. Further, in this case where $K \geq (\delta - 1)(x + 1)$, it holds that $\bar{\tau}(x) \leq \frac{K}{(\delta - 1)(x + 1)} \bar{\tau}(x)$. To summarize, because $\beta < \bar{\tau}(x)$, the feasible region $[\underline{z}, \bar{z}] \cup [\tilde{z}, \hat{z}]$ is non-empty if and only if $\beta \leq \min\{\frac{K}{((d-p)-(r-d)x)} - 1, \bar{\tau}(x)\}$. By definition, $\tau(x) = \min\{\frac{K}{((d-p)-(r-d)x)} - 1, \bar{\tau}(x)\}$. Because $K > \underline{K}$, it follows that $\tau(x) > 0$. Therefore, (9) is feasible if and only if $\beta \leq \tau(x)$, which is also sufficient and necessary for formulation (18) to be feasible.

Step 2. In this step, we solve for the optimal solution z^* . To do so, we first show that problem (9) is equivalent to the following problem:

$$\max \Pi_s(0, 0, z) \text{ s.t. } z \in [\underline{z}, \bar{z}]. \quad (10)$$

To do so, we consider two cases: $K < (\delta - 1)(x + 1)$ and $K \geq (\delta - 1)(x + 1)$. If $K < (\delta - 1)(x + 1)$, we have showed in Step 1 that the optimal solution z^* lies in the feasible region and satisfies $z^* \in [\underline{z}, K - ((d - p) - x(r - d))]$. If $K \geq (\delta - 1)(x + 1)$, it follows from Step 1 that for any β that satisfies $0 < \beta \leq \tau(x)$, the feasible region is $[\underline{z}, \hat{z}]$, where $\hat{z} = K \left(1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}\right) \geq \tilde{z}$. Note that when $z \in [\tilde{z}, \hat{z}]$, from the definition in (15), we have $\Pi_s(0, 0, z) = K - z$, which decreases in z . It follows that $\Pi_s(0, 0, \tilde{z}) > \Pi_s(0, 0, z)$ for all $z > \tilde{z}$. Therefore, when $K \geq (\delta - 1)(x + 1)$, to maximize $\Pi_s(0, 0, z)$, the optimal solution z^* should satisfy $z^* \in [\underline{z}, \tilde{z}]$, where $\tilde{z} = (\delta - 1)(x + 1) - ((d - p) - (r - d)x)$. To conclude, z^* must satisfy $z^* \in [\underline{z}, \min\{K, (\delta - 1)(x + 1)\} - ((d - p) - x(r - d))]$, or equivalently, $z^* \in [\underline{z}, \bar{z}]$. Thus formulation (9) is equivalent to formulation (10). Next, we solve problem (10). Using the profit expression given in (15), and the fact that $\delta > 1$, we have $\frac{\partial^2 \Pi_s(0, 0, z)}{\partial z^2} = -\frac{2}{(\delta - 1)(x + 1)} < 0$. Hence, $\Pi_s(0, 0, z)$ is strictly concave in z . The solution to the first order condition $\frac{\partial \Pi_s(0, 0, z)}{\partial z} = 0$ is $\hat{z} = \frac{K}{2} - \frac{1}{2}((d - p) - (r - d)x)$. Therefore, the unique optimal subsidy z^* is given by

$$z^* = \begin{cases} \underline{z} & \text{if } \hat{z} < \underline{z}, \\ \hat{z} & \text{if } \underline{z} \leq \hat{z} \leq \bar{z}, \\ \bar{z} & \text{if } \bar{z} < \hat{z}, \end{cases}$$

or, equivalently, $z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\}$. Note that z^* is the unique optimal solution to problem (10), so z^* is the unique feasible, and thus optimal, solution to (18).

Step 3. Because $\bar{z} = \min\{K, (\delta - 1)(x + 1)\} - ((d - p) - x(r - d))$, we can rewrite z^* as $z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}_1, \bar{z}_2\}$, where $\bar{z}_1 = K - ((d - p) - x(r - d))$ and $\bar{z}_2 = (\delta - 1)(x + 1) - ((d - p) - x(r - d))$. Note $\frac{\partial \bar{z}}{\partial x} = -\beta(r - d) < 0$, $\frac{\partial \hat{z}}{\partial x} = \frac{r-d}{2} > 0$, $\frac{\partial \bar{z}_1}{\partial x} = r - d > 0$ and $\frac{\partial \bar{z}_2}{\partial x} = (\delta - 1) + (r - d) > 0$. Because \underline{z} , \hat{z} , \bar{z}_1 , and \bar{z}_2 are all continuous in x , z^* is continuous in x . Therefore, if $z^* = \underline{z}$, z^* decreases in x ; otherwise, z^* increases in x . It remains to show the conditions under which $z^* = \underline{z}$. In preparation, let $\tilde{x} = \bar{x} - \frac{K}{(2\beta+1)(r-d)}$ and $K_1 = (\bar{x} - \underline{x})(2\beta + 1)(r - d)$, and note that $\tilde{x} < \bar{x}$ by definition of \tilde{x} and because $r > d$. Note that $z^* = \underline{z}$ if and only if $\hat{z} \leq \underline{z}$, which holds if and only if $x \leq \tilde{x}$. Because $x \in (\underline{x}, \bar{x})$, $x \leq \tilde{x}$ can hold only if $\tilde{x} > \underline{x}$. It can be verified that $\tilde{x} > \underline{x}$ if and only if $K < K_1$. Note that $K_1 = (\bar{x} - \underline{x})(2\beta + 1)(r - d) \geq (d - p) - (r - d)\underline{x} > (d - p) - (r - d)x = \underline{K}$. Therefore, (i) if $K \geq K_1$, z^* increases in x on (\underline{x}, \bar{x}) ; (ii) if $\underline{K} < K < K_1$, $z^* = \underline{z}$ decreases in x on $(\underline{x}, \tilde{x}]$ and increases in x on (\tilde{x}, \bar{x}) . The result follows. \square

Proof of Theorem 1. First, we compare the performances of the two mechanisms in metrics I – V. By Corollary 2, we have $C^D = C(e^*, s^*, 0) > 0$, and under mechanism [I], $C^I = C(0, 0, z^*) = 0$. Hence, $C^I = 0 < C^D$. Next, using the expressions for e^* and s^* given in Proposition 1, and the expression for z^* given in Proposition 2, it can be shown that $D_m(0, 0, z^*) - D_m^0 \geq \beta D_m^0 = D_m(e^*, s^*, 0) - D_m^0$, or equivalently, $D_m^I \geq D_m^D$. Because $D_m^I \geq D_m^D$, from (13) and (14) we immediately obtain $\Pi_p^I \geq \Pi_p^D$ and $\Pi_r^I \geq \Pi_r^D$, respectively. To compare the commuter welfare in mechanism [D] and mechanism [I], it is easy to check that $W^D = W^0$, because constraint (16c) is binding in equilibrium. Further, $W^I - W^0 = W(0, 0, z^*) - W^0 = \frac{z^*(2(d-p)-2(r-d)x+z^*)}{2(\delta-1)(x+1)}$, which satisfies $W^I - W^0 > 0$ if and only if $z^* > 0$. To see that $z^* > 0$ holds, let us first define $\gamma_1 = (\delta - 1)(x + 1) > 0$ and $\gamma_2 = (d - p) - x(r - d) > 0$. Then $\underline{z} = \beta\gamma_2 > 0$, $\hat{z} = \frac{K-\gamma_2}{2}$, $\bar{z} = \min\{K - \gamma_2, \gamma_1 - \gamma_2\}$ and $\tau(x) = \min\{\frac{K}{\gamma_2} - 1, \frac{\gamma_1}{\gamma_2} - 1\}$. Because $K > \underline{K} = \gamma_2$, it follows $\hat{z} = \frac{K-\gamma_2}{2} > 0$ and $\bar{z} = \min\{K - \gamma_2, \gamma_1 - \gamma_2\} > 0$. Therefore, $z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\} > 0$, and thus $W^I > W^0 = W^D$.

For the enterprise's profit, using the definition in (15), we have $\Pi_s^I - \Pi_s^D = \frac{\hat{z}^2 + K\gamma_2 - (z^* - \hat{z})^2}{\gamma_1} - \frac{K\gamma_2(\beta+1)}{\gamma_1}$. Define $\tilde{\tau}(x) = \frac{\hat{z}^2 - (z^* - \hat{z})^2}{K\gamma_2}$. Then $\Pi_s^I < \Pi_s^D$ holds if and only if $\beta > \tilde{\tau}(x)$. Note mechanism [I] is feasible if and only if $\beta \in (0, \tau(x)]$, by Proposition 2. To see there exists β such that both $\beta > \tilde{\tau}(x)$ holds and mechanism [I] is feasible, it suffices to show $0 \leq \tilde{\tau}(x) \leq \tau(x)$ for all x . Because $z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\} \in \{\underline{z}, \hat{z}, \bar{z}\}$, we shall prove $0 \leq \tilde{\tau}(x) \leq \tau(x)$ for all x by considering three cases: $z^* = \underline{z}$, $z^* = \hat{z}$ and $z^* = \bar{z}$. Because $0 < \beta \leq \tau(x)$, it follows that $\underline{z} \leq \bar{z}$, and thus the three cases correspond to $\hat{z} \leq \underline{z}$, $\underline{z} < \hat{z} \leq \bar{z}$, and $\bar{z} < \hat{z}$.

Case 1: Suppose $\hat{z} \leq \underline{z}$. Then $z^* = \underline{z}$. It follows that $\tilde{\tau}(x) = \frac{\hat{z}^2 - (\underline{z} - \hat{z})^2}{K\gamma_2} = \frac{2\underline{z}\hat{z} - \underline{z}^2}{K\gamma_2} \leq \frac{\underline{z}^2}{K\gamma_2} \leq \frac{\bar{z}^2}{K\gamma_2} = \frac{(\min\{K, \gamma_1\} - \gamma_2)^2}{K\gamma_2} < \frac{(\min\{K, \gamma_1\} - \gamma_2) \min\{K, \gamma_1\}}{K\gamma_2} = \tau(x) \min\{1, \frac{\gamma_1}{K}\} \leq \tau(x)$. Further, because $\beta \leq \tau(x) = \min\{\frac{K}{\gamma_2} - 1, \frac{\gamma_1}{\gamma_2} - 1\}$, it follows that $\beta\gamma_2 \leq K - \gamma_2$, or equivalently, $z^* = \underline{z} \leq 2\hat{z}$. Thus $\tilde{\tau}(x) = \frac{\hat{z}^2 - (\underline{z} - \hat{z})^2}{\gamma_1\gamma_2} \geq 0$. Therefore, in this case, $0 \leq \tilde{\tau}(x) \leq \tau(x)$ holds for all x .

Case 2: Suppose $\underline{z} < \hat{z} \leq \bar{z}$. Then $z^* = \hat{z}$ and $\tilde{\tau}(x) = \frac{\hat{z}^2}{K\gamma_2} \leq \frac{\bar{z}^2}{K\gamma_2} < \tau(x)$. Further, it is straightforward to verify that $\tilde{\tau}(x) > 0$. Therefore, $0 \leq \tilde{\tau}(x) \leq \tau(x)$ holds for all x .

Case 3: Suppose $\bar{z} < \hat{z}$. Then $z^* = \bar{z}$. Because $\bar{z} = \min\{\gamma_1 - \gamma_2, K - \gamma_2\}$ and $\hat{z} = \frac{K-\gamma_2}{2} < K - \gamma_2$, $\bar{z} \leq \hat{z}$ implies $\bar{z} = \gamma_1 - \gamma_2 < K - \gamma_2$, or equivalently, $\gamma_1 < K$. It follows that $\tau(x) = \frac{\gamma_1}{\gamma_2} - 1$, and thus $\tilde{\tau}(x) = \frac{2\underline{z}\bar{z} - \bar{z}^2}{K\gamma_2} = \frac{2(\gamma_1 - \gamma_2)\frac{K-\gamma_2}{2} - (\gamma_1 - \gamma_2)^2}{K\gamma_2} = \tau(x)(1 - \frac{\gamma_1}{K}) < \tau(x)$. It is straightforward to verify that $\tilde{\tau}(x) > 0$. Therefore, $0 \leq \tilde{\tau}(x) \leq \tau(x)$ holds for all x .

To summarize the three cases above, (i) if $\beta \leq \tilde{\tau}(x)$, then $\Pi_s^I \geq \Pi_s^D$; (ii) if $\beta \in (\tilde{\tau}(x), \tau(x)]$, then $\Pi_s^I < \Pi_s^D$. The result follows.

Proof of Proposition 3. Similar to the proof of Proposition 1, the proof proceeds in two steps. In Step 1, we find the optimal solutions (e^*, s^*) . In Step 2, we prove the comparative statics results.

Step 1. To solve for (e^*, s^*) , we consider two cases: $(D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)})$ and $(D_m(e, s, 0) = 1)$. We then show that the optimal commuter welfare is higher in the latter case.

Case 1: Suppose $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$, which is equivalent to

$$e + s \leq (\delta - 1)(x + 1) - ((d - p) - (r - d)x). \quad (11)$$

The proof proceeds in two steps. First, we construct an upper bound for the optimal value of the objective function (20a) when adding (11) as an additional constraint to problem (20); second, to show optimality, we construct a feasible solution that attains this bound. First, because $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$, we can apply (4) and rewrite constraint (20c) as

$$C(e, s, 0) = (W(e, s, 0) - W^0) + \frac{(e + s)^2}{2(\delta - 1)(x + 1)} = 0,$$

which implies any feasible (e, s) should satisfy $W(e, s, 0) - W^0 = -\frac{(e+s)^2}{2(\delta-1)(x+1)}$. Next, constraint (20b) can be rewritten as $e + s \geq \beta((d - p) - (r - d)x)$. Note $\beta((d - p) - (r - d)x) > 0$ because $x \in (\underline{x}, \bar{x})$. Therefore,

$$W(e, s, 0) - W^0 = -\frac{(e + s)^2}{2(\delta - 1)(x + 1)} \leq -\frac{(\beta((d - p) - x(r - d)))^2}{2(\delta - 1)(x + 1)},$$

which holds with equality if and only if constraint (20b) is binding. Next, we prove that the upper bound is attainable. We first solve for (e, s) such that constraint (20b) is binding and constraint (20c) is satisfied:

$$\begin{aligned} D_m(e, s, 0) - D_m^0 &= \beta D_m^0 \\ C(e, s, 0) &= 0. \end{aligned} \quad (12)$$

Solving the above set of equations yields the unique solution

$$\begin{aligned} e_2 &= \frac{\beta(\beta + 1)((d - p) - (r - d)x)^2}{(\delta - 1)(x + 1)} \\ s_2 &= \frac{\beta((d - p) - (r - d)x)((\delta - 1)(x + 1) - (\beta + 1)((d - p) - (r - d)x))}{(\delta - 1)(x + 1)}. \end{aligned}$$

Using similar arguments in proof of Proposition 1, it is plain to verify that (e_2, s_2) satisfies the additional constraint (11) and it's feasible to problem (20), so (e_2, s_2) is the unique optimal solution to formulation (20) when restricting $D_m(e, s, 0) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)}$. The corresponding optimal commuter welfare increment is $\Delta W(e_2, s_2, 0) = -\frac{(\beta((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)}$.

Case 2: Suppose $D_m(e, s, 0) = 1$, which is equivalent to $e + s \geq (\delta - 1)(x + 1) - ((d - p) - (r - d)x)$. It

follows that $D_d(e, s, 0) = 0$. Then the budget neutrality constraint (20c) can be rewritten as

$$C(e, s, 0) = sD_m(e, s, 0) - eD_d(e, s, 0) = s = 0. \quad (13)$$

From $D_m(e, s, 0) = 1$, it follows that $e \geq (\delta - 1)(x + 1) - ((d - p) - (r - d)x)$. From the definition in (12), the commuter welfare increment is

$$\Delta W(e, s, 0) = -\frac{((\delta - 1)(x + 1) - ((d - p) - x(r - d)))^2}{2(\delta - 1)(x + 1)}. \quad (14)$$

Because $0 < \beta < \bar{\tau}(x)$, it can be verified that for all (e, s) that satisfies $D_m(e, s, 0) = 1$, $\Delta W(e, s, 0) < \Delta W(e_2, s_2, 0)$. Therefore, the unique optimal solution to (20), denoted by (e^*, s^*) , must satisfy $D_m(e^*, s^*, 0) = \frac{(d-p)-(r-d)x+e^*+s^*}{(\delta-1)(x+1)}$ and $(e^*, s^*) = (e_2, s_2)$.

Step 2. Next, we prove statements (i)-(iii) in order. (i). We first take the first derivative of e^* with respect to x and β . Because $\delta > 1$, $0 < x < \bar{x}$ and $r > d > p > 0$, it follows that $\frac{\partial e^*}{\partial \beta} = \frac{(2\beta+1)((d-p)-(r-d)x)^2}{(\delta-1)(x+1)} > 0$. Next, $\frac{\partial e^*}{\partial x} = \frac{\beta(\beta+1)((x+1)^2(r-d)^2-(r-p)^2)}{(\delta-1)(x+1)^2}$. Because $r - p = (d - p) + (r - d) > (r - d)(x + 1)$, it follows that $\frac{\partial e^*}{\partial x} < 0$. Therefore, e^* strictly increases in β and strictly decreases in x . (ii). First, note that s^* is concave in x : $\frac{\partial^2 s^*}{\partial x^2} = -\frac{2\beta(\beta+1)(p-r)^2}{(\delta-1)(x+1)^3} < 0$. Then, to show the result, it suffices to check the first derivatives at the boundaries: $\frac{\partial s^*}{\partial x} \Big|_{x=\bar{x}}$ and $\frac{\partial s^*}{\partial x} \Big|_{x=\underline{x}}$. If $\delta < \delta_1 = 1 + d - p$, then $\underline{x} = \frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}$, and $\frac{\partial s^*}{\partial x} \Big|_{x=\underline{x}} = \beta((\beta + 1)(\delta - 1) + (2\beta + 1)(r - d)) > 0$; if $\delta \geq \delta_1$, then $\underline{x} = 0$, and $\frac{\partial s^*}{\partial x} \Big|_{x=\underline{x}} = \frac{\beta((\beta+1)(d-p)(2r-d-p)-(\delta-1)(r-d))}{\delta-1}$. If $\delta < \delta_3 = \frac{(\beta+1)(d-p)(2r-d-p)}{r-d} + 1$, $\frac{\partial s^*}{\partial x} \Big|_{x=\underline{x}} > 0$; if $\delta \geq \delta_3$, $\frac{\partial s^*}{\partial x} \Big|_{x=\underline{x}} \leq 0$. Notice $\delta_3 - \delta_1 = \frac{(d-p)(\beta(2r-d-p)+r-p)}{r-d} > 0$, or $\delta_3 > \delta_1$. We conclude that if $\delta \geq \delta_3$, $\frac{\partial s^*}{\partial x} \Big|_{x=\underline{x}} \leq 0$, and if $1 < \delta < \delta_3$, $\frac{\partial s^*}{\partial x} \Big|_{x=\underline{x}} > 0$. Therefore, by continuity of $\frac{\partial s^*}{\partial x}$ in x , it follows that if $\delta \geq \delta_3$, s^* decreases in $x \in (\underline{x}, \bar{x})$, and if $\delta < \delta_3$, there exists $\tilde{x} \in (\underline{x}, \bar{x})$ such that s^* strictly increases in x on (x, \tilde{x}) and strictly decreases in x on (\tilde{x}, \bar{x}) . (iii). Note $\frac{\partial s^*}{\partial \beta} = \frac{1}{(\delta-1)(x+1)}(((d-p)-(r-d)x)((\delta-1)(x+1)-(2\beta+1)((d-p)-(r-d)x)))$. Then if $\beta < \frac{(\delta-1)(x+1)}{2((d-p)-(r-d)x)} - \frac{1}{2} = \frac{\bar{\tau}(x)}{2}$, $\frac{\partial s^*}{\partial \beta} > 0$, and if $\beta \geq \frac{\bar{\tau}(x)}{2}$, $\frac{\partial s^*}{\partial \beta} \leq 0$. Therefore, s^* increases in $\beta \in (0, \frac{\bar{\tau}(x)}{2})$ and decreases in $\beta \in [\frac{\bar{\tau}(x)}{2}, \bar{\tau}(x)]$. \square

Proof of Corollary 3. Note $e^* - s^* = \beta((d - p) - (r - d)x) \left(\frac{2(\beta+1)((d-p)-(r-d)x}{(\delta-1)(x+1)} - 1 \right)$. Because $x \in (\underline{x}, \bar{x})$, $(d - p) - (r - d)x > 0$. Therefore, $e^* \geq s^*$ if and only if $\frac{2(\beta+1)((d-p)-(r-d)x}{(\delta-1)(x+1)} - 1 \geq 0$. It follows that $e^* \geq s^*$ if and only if $x \leq \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)(2(\beta+1)(r-d)+\delta-1)}$. Defining $\hat{x} = \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)(2(\beta+1)(r-d)+\delta-1)}$ yields the main result. Following a parallel argument to the proof of Corollary 1, define $\bar{\delta} = 2(\beta + 1)(d - p) + 1$, which satisfies $\bar{\delta} > 1$. It can then be shown that $s^* > e^*$ for all $x \in (\underline{x}, \bar{x})$ if and only if $\hat{x} \leq \underline{x}$, which holds if and only if $\delta \geq \bar{\delta}$. \square

Proof of Corollary 4. By plugging in the optimal solutions (e^*, s^*) in Proposition 3, the change in commuter welfare is given by

$$\Delta W(e^*, s^*, 0) = W(e^*, s^*, 0) - W^0 = -\frac{\beta^2((d-p)-(r-d)x)^2}{2(\delta-1)(x+1)} < 0.$$

Note $\frac{\partial \Delta W(e^*, s^*, 0)}{\partial \beta} = -\frac{\beta((d-p)-x(r-d))^2}{(\delta-1)(x+1)} < 0$, and thus $\Delta W(e^*, s^*, 0)$ strictly decreases in β . Next, note $\frac{\partial \Delta W(e^*, s^*, 0)}{\partial x} = \frac{\beta^2((r-p)^2-(x+1)^2(r-d)^2)}{2(\delta-1)(x+1)^2}$. Because $x \in (\underline{x}, \bar{x})$, $0 < (x+1)(r-d) < (r-p)$. It follows that $\frac{\partial \Delta W(e^*, s^*, 0)}{\partial x} > 0$, and thus $\Delta W(e^*, s^*, 0)$ strictly increases in x . \square

Proof of Proposition 4. Based on the proof of Proposition 2, constraints (18b), (18d) and (18e), which are equivalent to constraints (21b), (21d) and (21e), uniquely define a feasible and thus optimal solution $z^* = \min\{\max\{z, \hat{z}\}, \bar{z}\}$. Because $C(0, 0, z^*) = 0$, it follows that z^* is also the unique feasible and thus optimal solution to formulation (21). To show that $W(0, 0, z) > W^0$, note that because $z^* > 0$, using the definition in (12), we have $W(0, 0, z) - W^0 = \frac{z(2(d-p)-2(r-d)x+z)}{2(\delta-1)(x+1)} > 0$. \square

Proof of Theorem 2. First, we compare the performance of mechanisms [D-A] and [I-A] in metrics I – V. By constraints (20c) and (21c), we have $C^I = C^D = 0$. Next, using the expressions for e^* and s^* given in Proposition 3, and the expression for z^* given in Proposition 4, it can be shown that $D_m(0, 0, z^*) - D_m^0 \geq \beta D_m^0 = D_m(e^*, s^*, 0) - D_m^0$, or equivalently, $D_m^I \geq D_m^D$. Because $D_m^I \geq D_m^D$, from (13) and (14) we immediately obtain $\Pi_p^I \geq \Pi_p^D$ and $\Pi_r^I \geq \Pi_r^D$, respectively. Recall from Corollary 4 and Proposition 4 that, $W(e^*, s^*, 0) - W^0 < 0$ and $W(0, 0, z^*) - W^0 > 0$, and thus $W^D < W^0 < W^I$. For metric VI, using definition in (15), we have $\Pi_s^I - \Pi_s^D = \frac{\hat{z}^2 + K((d-p)-x(r-d)) - (z^* - \bar{z})^2}{(\delta-1)(x+1)} - \frac{K((d-p)-x(r-d))(\beta+1)}{(\delta-1)(x+1)}$, which is equal to the enterprise's profit difference between mechanisms [D] and [I]. Following the proof of Theorem 1, if $\beta \leq \tilde{\tau}(x)$, then $\Pi_s^I \geq \Pi_s^D$, and if $\beta \in (\tilde{\tau}(x), \tau(x)]$, then $\Pi_s^I < \Pi_s^D$. The result follows. \square